Quick Note on Signal to Noise Ratio

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Astronomers mostly use continuum **magnitudes**: ABMAG for target objects, and similarly we use ABMAG/square arcsec for night sky continuua.
Physics mostly use **photon fluxes** N_λ in photons/m^2.sec.μm for target objects, or in photons/m^2.sec.μm.square arcsec for night sky continuua.
Connection between these: $N_\lambda = \mathrm{dex}(10.742-0.4*\mathrm{ABMAG})/\lambda_{\mu m}$

Assumptions & definitions for this Note:
Telescope effective area A (sq meters) including quantum efficiency losses, etc;
Bandwidth B also often written as $\Delta\lambda$ (expressed in microns);
Exposure time T, in seconds;
Filter transmission Fgal for the average continuum light from a target galaxy
Filter transmission Fsky but now weighted by the night sky spectrum
Ngal = in-band photon continuum from a given target galaxy
Ωgal = sky area, square arcsec, needed to capture most of the galaxy
Nsky = in-band photon lines + continuum from night sky
Ωref = sky area, square arcsec, of a big reference patch for sky subtraction.
Goal of this exercise is to determine the signal to noise ratio of the galaxy.

For an exposure of time duration T, the accumulated photoelectron count is
$\mathrm{Cgal} = A\cdot B\cdot T\cdot(\mathrm{Fgal}\cdot\mathrm{Ngal} + \mathrm{Fsky}\cdot\mathrm{Nsky}\cdot\Omega\mathrm{gal})$ for the galaxy with its sky, and
$\mathrm{Cref} = A\cdot B\cdot T\cdot(\mathrm{Fsky}\cdot\mathrm{Nsky}\cdot\Omega\mathrm{ref})$ for the reference sky patch.
These counts "C" obey independent arrival Poisson statistics: var = mean.
The background-subtracted galaxy-alone estimator $G = \mathrm{Cgal} - \mathrm{Cref}\cdot\Omega\mathrm{gal}/\Omega\mathrm{ref}$.;
its mean is $<G> = A\cdot B\cdot T\cdot\mathrm{Fgal}\cdot\mathrm{Ngal}$, whence $<\mathrm{Ngal}>=<G>/A\cdot B\cdot T\cdot\mathrm{Fgal}$.
Its variance is $\mathrm{var}(G) = A\cdot B\cdot T\cdot(\mathrm{Fgal}\cdot\mathrm{Ngal}+ \mathrm{Fsky}\cdot\mathrm{Nsky}\cdot\Omega\mathrm{gal}\cdot[1+\Omega\mathrm{gal}/\Omega\mathrm{ref}])$

Now adopt two important limits, namely (1) a large reference area is available for sky subtraction $\Omega\mathrm{ref} \gg \Omega\mathrm{gal}$, and also (2) the galaxy is much fainter than the sky $\mathrm{Fgal}\cdot\mathrm{Ngal} \ll \mathrm{Fsky}\cdot\mathrm{Nsky}\cdot\Omega\mathrm{gal}$. With these two limits, the variance simplifies to
$\mathrm{var}(G) = A\cdot B\cdot T\cdot\mathrm{Fsky}\cdot\mathrm{Nsky}\cdot\Omega\mathrm{gal}$.

The signal-to-noise ratio is $\mathrm{SNR} = <G>/\mathrm{sqrt}(\mathrm{var}(G))$ which in these two limits is
$\mathrm{SNR} = \sqrt{(ABT)}\cdot\mathrm{Fgal}\cdot\mathrm{Ngal}/\sqrt{(\mathrm{Fsky}\cdot\mathrm{Nsky}\cdot\Omega\mathrm{gal})}$

We see that good SNR for a given Ngal requires large telescopes, large bandwidths, high filter transmission for the galaxy continuum Fgal, good seeing = small Ωgal, dark skies, and low sky filter transmission. The **filter figure of merit** here is...
$\mathrm{FFOM} = \mathrm{Fgal}/\sqrt{\mathrm{Fsky}}$. If Fgal attenuates a factor of two, the sky must attenuate a factor of 4 to get the same FFOM.