Note on Estimating Mean Time to Failure "MTTF"

DESI 4191-v3, 19 Oct 2018, M.Lampton UCB SSL

"To a man with a hammer, everything looks like a nail."
-Abraham Maslow

Prelude: Last week, in connection with the DESI GFA qualification testing, I was asked did I know anything about how to estimate failure rates from limited data. I replied "Nothing but that famous bathtub curve." But I later recalled that I was once an X-ray astronomer, and X-ray astronomers draw inferences from Poisson limited data, namely the arrivals of a few statistically random independent photon events. Just like observing a few failures in a life test program! So I wrote this short note based on that background[1].

Viewpoint 1: Theorist. This is the easy case. A theorist knows, or models, the truth and then with utter certainty can predict the probabilities of various measurement outcomes. For independent random event arrivals, those probabilities are the Poisson distribution,

$$Prob(n : \lambda) = \frac{\lambda^n \, e^{-\lambda}}{n!} \tag{1}$$

This expression gives the probability of seeing exactly n events (always an integer!) where the true (infinite ensemble) mean number is λ. This λ is almost never an integer, but in the theorist view is exactly known or modelled.

Viewpoint 2: Observer. This is the hard case. An observer knows he/she has observed n events, always an integer, and wants to infer what the true mean λ might be. But owing to the arrival statistics, λ can be inferred only to lie within bounds limited by a confidence region. He/she could choose a broad high-confidence region with consequent poor knowledge of λ, or a narrower one with tighter limits on λ but less confidence that it is correct. For inference, the term *relative likelihood* with symbol $\mathcal{L}$ is used:

$$\mathcal{L}(\lambda : n) = \frac{\lambda^n \, e^{-\lambda}}{n!} \tag{2}$$

This is exactly the same as equation 1, but its meaning is utterly different: n is no longer a statistic, it is a fact. Here λ is no longer given, but is completely unknown. $\mathcal{L}$ compares the likelihood of one possible λ against another λ. This switcheroo derives from Bayes' theorem for the simplest case of a "uniform prior" meaning we have no initial knowledge of the true values of λ. Its meaning is simply that among competing theories, those that predict what we observed are plausibile, while those that predict outcomes that are rather different are implausible. Using a continuous range of possible λ values, we can calculate the confidence that any specified interval (A, B) contains the true λ by integrating $\mathcal{L}$ over that interval.

[1]Lampton,M, Margon,B, andBowyer,S, "Parameter Estimation in Xray Astronomy," Ap.J. 208 (1976).

Extension to Failure Rate We don't care if the true failure rate is very low, but we care strongly if it is high. So we seek a one-sided confidence region: with a stated confidence, we want to claim that λ is less than some maximum bound B.

EXCEL® offers the incomplete gamma function `GAMMADIST(B,NOBS+1,1,TRUE)` that evaluates the cumulative normalized integral of $\mathcal{L}$ over the range $0 < \lambda < B$. This is exactly what we want: it tells us the confidence that the true population failure rate is below a claimed bound B, given an observed number N_{obs} of random failure events.

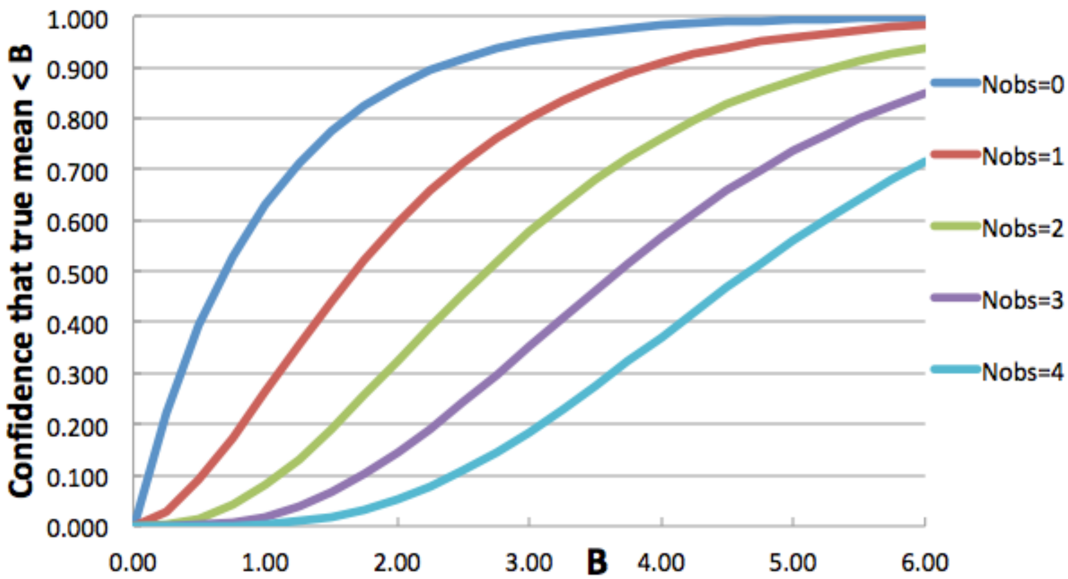

Figure 1: Plot shows the confidence that the true failure mean per campaign is less than B, given how many failures N_{obs} were observed in that campaign. If that campaign spans U units and Y years, the failure rate per unit is less than B/UY failures per year per unit, and the MTTF is greater than UY/B years per unit.

Example, long test: We run ten units for one year, and observe (say) two failures. The boss insists we work to 68% confidence. We conclude that our true unit failure rate is no greater than B/UY where (from the chart) $B = 3.5$, UY (from the test) =10, and therefore our true failure rate is no greater than $B/UY = 0.35$ failures per year per unit, or $MTTF > UY/B = 3$ years. These puppies last, on average, more than 3 years. Good!

Example, short test: We run ten units for 0.1 years and observe (say) two failures. At 68% confidence, again we have $B = 3.5$ but $UY = 1$ unit-years so $B/UY = 3.5$ failures per year per unit, or $MTTF > UY/B = 0.3$ years. Not so useful.

Example, short test: We run ten units for 0.1 years and observe (say) zero failures. Hooray! Now we have from the chart $B = 1.0$ and from the test $UY = 1.0$ so with 68% confidence, our failure rate is less than $B/UY = 1.0$ failures per year per unit, and that $MTTF > UY/B = 1.0$ years. Maybe OK; depends on mission criticality.