Note: Estimating Mean Time to Failure "MTTF"

rev 1, 8 Oct 2018, M.Lampton UCB SSL

"To a man with a hammer, everything looks like a nail."
-Abraham Maslow

Prelude: Last week, in connection with the DESI GFA qualification testing, I was asked did I know anything about how to estimate failure rates from limited data. I replied "Nothing but that famous bathtub curve."

But in the shower this morning, I recalled that I was once an X-ray astronomer, and X-ray astronomers draw inferences from Poisson limited data, namely the arrivals of a few statistically random independent photon events. Just like observing a few failures in a test rig! So I have assembled this short note based on that background[1].

Disclaimer: I am not a reliability engineer. My stuff needs vetting or correction.

Viewpoint 1: Theorist This is the easy case. A theorist knows, or models, the truth and then with utter certainty can predict the probabilities of various measurement outcomes. For independent random arrivals, that result is the Poisson distribution,

$$Prob(n) = \frac{\lambda^n \, e^{-n}}{n!} \tag{1}$$

This expression gives the probability of seeing exactly n events (always an integer!) where the true (infinite ensemble) mean number is λ. This λ is almost never an integer, but in the theorist view is exactly known or modelled.

Viewpoint 2: Observer This is the hard case. An observer knows he/she has observed n events, always an integer, and wants to infer what the true mean λ might be. But owing to the arrival statistics, λ can be inferred only to lie within bounds limited by a confidence region. He/she could choose a wide confidence region with consequent poor knowledge of λ, or a narrower one with tighter limits on λ but less confidence that it is correct. For confidences, the term *likelihood* with symbol $\mathcal{L}$ is used:

$$\mathcal{L}(\lambda,\ n) = \frac{\lambda^n \, e^{-n}}{n!} \tag{2}$$

Although $\mathcal{L}$ is like a probability and has both a density and can be integrated over a range of λ, it is unlike a probability in requiring normalization. It derives from Bayes' theorem for the simplest case of a "uniform prior" meaning we have no initial knowledge of the true values of λ. So, we normalize $\mathcal{L}$ in such a way that when integrated over all possible values of λ, we get 1; after all λ has to be in there somewhere!

[1] Lampton,M, Margon,B, and Bowyer,S, "Parameter Estimation in Xray Astronomy," Ap.J. 208 (1976).

Extension to Failure Rate This half the observer situation. We don't care if the true failure rate is very low, but we care strongly if it is high. So we seek a one-sided confidence region for λ.

EXCEL® offers the incomplete gamma function *GAMMADIST(X,N+1,1,TRUE)* that evaluates the cumulative normalized integral of $\mathcal{L}$ over the range $0 < \lambda < X$. This is exactly what we want: it tells us the confidence that the true failure rate is lower than a given maximum failure rate X, given some observed number N_{obs} of random failure events.

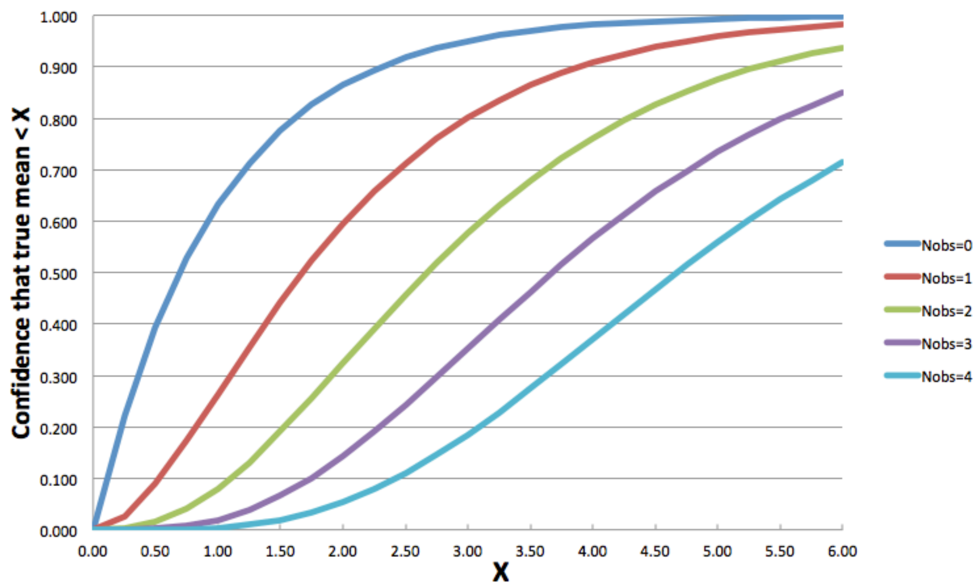

Figure 1: The likelihood that the true failure mean per campaign is less than X, given how many failures N_{obs} were observed in that campaign. If that campaign spans U units and Y years, the failure rate per unit is X/UY failures per year per unit, and the MTTF is UY/X years per unit.

Example, long test: We run ten units for one year, and observe (say) two failures. The boss insists we work to 68% confidence. We conclude that our true "infinite span" unit failure rate is no greater than X/UY where (from the chart) $X = 3.5$, $UY = 10$ and therefore $X/UY = 0.35$ failures per year per unit, or $MTTF > UY/X = 3$ years. Good!

Example, short test: We run ten units for 0.1 years and observe (say) two failures. At 68% confidence, again we have $X = 3.5$ but $UY = 1$ unit-years so $X/UY = 3.5$ failures per year per unit, or $MTTF > UY/X = 0.3$ years. Not so good!

Example, short test: We run ten units for 0.1 years and observe (say) zero failures! Now we have $X = 1.2$ so $X/UY = 1.2$ failures per year per unit, or $MTTF > UY/X = 0.8$ years. Maybe OK.

Question: I'm new at this. Did I do it correctly?