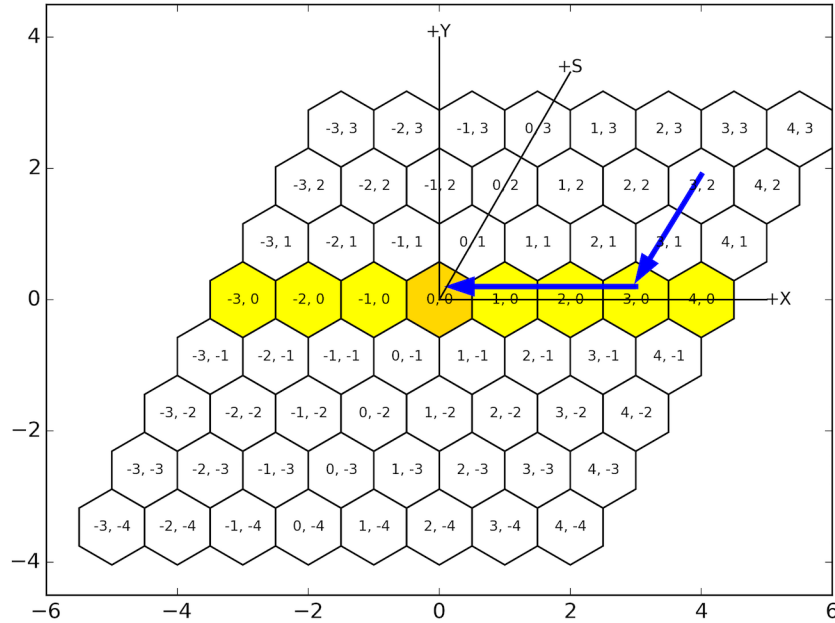


## Hexagonal Grids

M.Lampton UCB SSL 26 Feb 2018 Draft 3

**Purpose:** Given an infinite plane grid of hexagons and a location  $(x,y)$  in the global or laboratory frame, what are that point's local  $(x, y)$  coordinates within its enclosing hexagon? This question arises in evaluating the optical behavior of hexagonal array lenses.

**Coordinate System:** I consider an infinite grid of regular unit hexagons in their 'Pointy Top' orientation as shown in the figure below. Each has a horizontal column index  $m$  and a row index  $n$  that increases along the slant axis  $S$ . The  $m, n$  addresses are shown. The blue arrows are the two reduction steps.



**Rule 1:** The pattern is periodic on  $S$ ; adding any integer to  $S$  does not change the local  $(x,y)$ .

**Rule 2:** The pattern is periodic in  $X$ ; adding any integer to  $X$  does not change the local  $(x,y)$ .

**Python Implementation:**

```
root3 = math.sqrt(3.0)
unitSlant = np.array([0.5, 0.5*root3])
unitHoriz = np.array([1.0, 0.0])

def getXYcenter(m, n):
    return m*unitHoriz + n*unitSlant

def getTopBot(xy):
    a = math.fabs(xy[0])
    b = math.fmod(a, 1.0)
    arg = b if b<0.5 else 1.-b
    top = (1.-arg)/root3
    bot = (arg-1.)/root3
    return top, bot

def reduce(xy):
    top, bot = getTopBot(xy)
    n = 0
    while xy[1] > top:
        xy -= unitSlant
        top, bot = getTopBot(xy)
        n += 1
    while xy[1] < bot:
        xy += unitSlant
        top, bot = getTopBot(x)
        n -= 1
    m = int(round(xy[0]))
    xy[0] -= m
    return xy[0], xy[1], m, n
```