

Confocal Heat Pump

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1 Introduction

Most optical systems are reciprocal, meaning that their throughputs are the same in each direction. Indeed the optical reciprocity theorem is regarded as a mainstay connecting electromagnetism to thermodynamics.^{1 2 3 4} A familiar way of expressing this theorem is: "If I can see you, you can see me."

Unfortunately there are few studies of nonreciprocal optical systems and *their* thermodynamic properties. Can such systems be built, and can they deliver useful energy? In this short report, I present one such system invented long ago by persons unknown to me, which I call a *confocal heat pump* since it is based on ideas stemming from confocal ellipsoids.

2 Confocal Ellipsoids

I begin this discussion by illustrating in Figure 1 (below) a simple confocal ellipsoid. It has two symmetrical point foci, A and B. It has the remarkable property that all light or heat emanating from A lands on B, and conversely. It is therefore completely reciprocal, which is to say it obeys the optical reciprocity theorem.

In Figure 2 below I show another smaller confocal ellipsoid. Its foci are at the same locations, but its eccentricity is greater and its size is therefore smaller. For foci located at $x = \pm 1$, the semimajor axis p and semiminor axis q are related to its eccentricity e by

$$p = 1/e \tag{1}$$

$$q = \sqrt{p^2 - 1} \tag{2}$$

¹Stokes, G., Cambridge and Dublin Math J., v.4 p.1-14 1849.

²Helmholtz, H. von, Handbuch der physilogischen Optik, p.169, 1856.

³Kirchhoff, G., Ann. Phys. v.11, pp.275-301, 1860.

⁴Born, M., and Wolf, E., Principles of Optics, 7th ed, Cambridge, p.423, 1999.

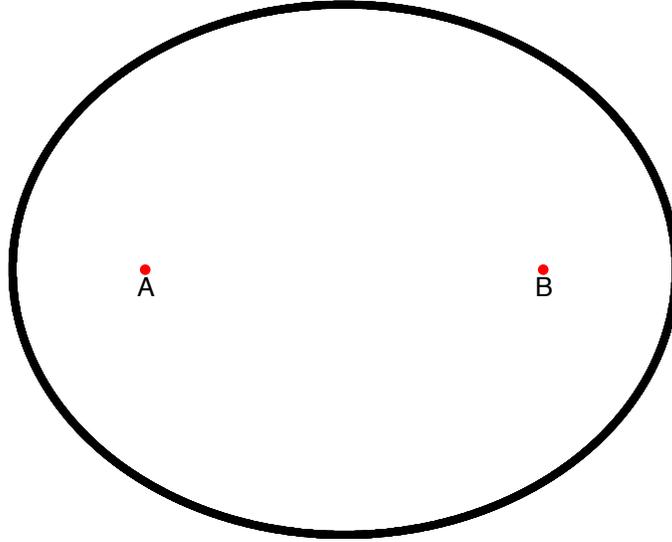


Figure 1: Side view of a concave ellipsoidal reflector, eccentricity= 0.6. Foci are the red dots. It is completely symmetrical: heat propagates equally well from B to A as from A to B.

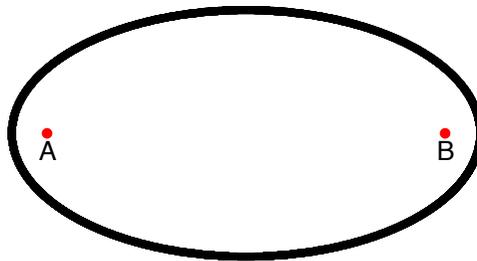


Figure 2: Like Figure 1, and with the same foci, but with eccentricity = 0.85. Also symmetrical. There are infinitely many other possibilities. Each is reciprocal.

3 Confocal Heat Pump

The preceding material shows that there exists a range of ellipsoidal mirror sizes that can couple one pair of tiny heat sources. To put this flexibility to work, I present the arrangement shown in Figure 3 below. Here the left portion of the reflector is the end cap of Figure 1, and is (as shown earlier) completely reciprocal. The right portion of the reflector is the end cap of Figure 2 and is (as shown earlier) also reciprocal. The symmetry is broken by inserting a section of a concave spherical mirror that joins these end caps. This zone is possible only if the sizes of the ellipsoids differ. There is a particularly tidy solution for that sphere's radius in which the spherical zone is just hidden from the right hand focus B while remaining centered on A. For the ellipticities shown here, its radius is 1.1065. For this arrangement, 100% of heat from B lands on A, cooling B, but only $\sim 80\%$ of heat from A lands on B. The remainder returns to A, reheating A.

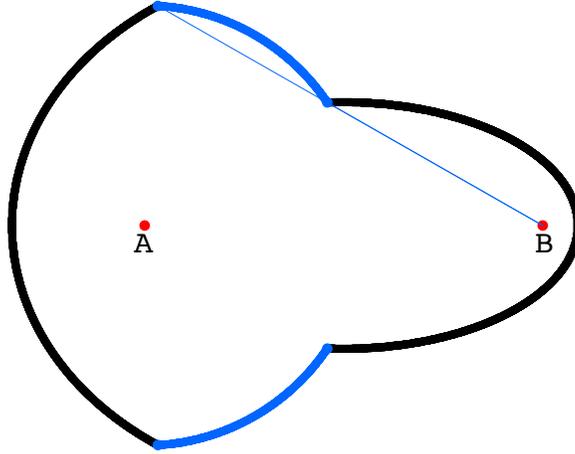


Figure 3: A confocal heat pump is produced by joining parts of two confocal ellipsoids having different eccentricities. A spherical mirror zone (blue) centered on focus A spans their gap. For each pair of possible ellipsoid eccentricities, there is a radius that allows the sphere zone surface to be exactly hidden from B yet fully visible to A. An edge ray is shown connecting the two ellipsoid edges to focus B.

4 Applications

With thermal blackbody source spectra and an 80/100 asymmetry, there is a natural limit to the temperature ratio that can be generated with this arrangement, namely $T_A/T_B = 0.8^{-1/4}$ or about an absolute temperature ratio of 1.05. Not big, but such a performance level could have many practical uses because around 300K this is a 15K available temperature difference.

In the winter: If the cold node B were heat sunk to the exterior of a house, its warm node A would run considerably warmer than the outside air temperature. The nonreciprocal behavior of the optic would extract heat from the outdoor environment and boost its temperature for indoor use. Unlike ordinary furnaces, this heat pump requires no energy. It could find useful application in polar areas where energy is expensive or unavailable.

In the summer: If the warm node A were heat sunk to the exterior of a house, its cold node B would run considerably cooler than the outside air temperature. Thus some useful measure of cooling could be achieved. Unlike ordinary air conditioning systems, this one requires no external power, and could be nicely suited to poorly developed desert areas where energy is expensive or unavailable.

Powering a heat engine: If a heat engine were to be connected between the confocal heat pump nodes, with its boiler at A and its condenser at B, it would produce useful work forever. Because energy is conserved, external heat would have to be supplied continually from an exterior source to compensate for energy taken from the system, but this could be ambient heat from any outdoor environment.

Cryo-refrigeration: (Suggested by P. Jelinsky) If the heat engine setup were to be disconnected from its external heat source, the entire system would grow colder until its working fluid freezes. Until that point however it would serve as a cryorefrigerator, removing heat

from a cold load at B and delivering that heat energy as useful work. This is an improvement on usual cryorefrigerators which require external power to deliver their cooling, and produce waste heat as an unwanted byproduct. However this application may well be impractical owing to the steep T^4 power transfer law that radiatively coupled systems obey: as the cryosystem cools, the rate at which heat can be extracted falls rapidly and cooldown times may be uselessly long.

5 Questions for Classroom Discussion

Q: A *first law* perpetual motion machine violates the First Law of Thermodynamics. Does this confocal heat pump claim to be a first law machine?

Q: A *second law* perpetual motion machine violates the Second Law of Thermodynamics. Does this confocal heat pump claim to be a second law machine?

Q: This confocal heat pump claims to be nonreciprocal. Is it? Why or why not?

Q: Are non-reciprocal devices ever possible? For one introduction see Lampton, M., "Transmission Matrices in Electroacoustics," *Acustica* v.39 no 4, pp.239-251, 1978.

Q: What is a Faraday rotator? Is it reciprocal? Is it a second law machine?

Q: What is a circulator? Is it reciprocal? Is it a second law machine?

Q: What is a one-way mirror? Is it reciprocal? Is it a second law machine?

Q: What is a cold mirror? Is it reciprocal? Is it a second law machine?

Q: What is a hot mirror? Is it reciprocal? Is it a second law machine?