

## LYMAN ALPHA AND X-RAY EMISSIONS DURING A SMALL SOLAR FLARE

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## ABSTRACT

A rocket instrumented to measure Lyman alpha and X-rays was fired while a small flare was in progress on June 20, 1956. The rocket reached peak altitude about ten minutes after the flare was first seen visually. An unusually high X-ray flux was observed extending to a short wavelength limit of 3A. Although the flare was still visible in H $\alpha$ , Lyman alpha was not appreciably different from normal.

Of all the types of activity observed on the sun, the solar flare is the phenomenon which appears to have the most direct effect at the earth. One of the most striking consequences of a flare is an immediate increase in ionization density in the *D* layer of the ionosphere with resultant radio fadeout (SID), sudden increase in atmospherics (SEA), and sudden phase anomaly in reflected long-wave radio signals (SPA). It is now well established that normal *D* layer is caused by the hydrogen Lyman  $\alpha$  line always present in the solar emission spectrum. This line is unable to ionize the primary atmospheric constituents (O<sub>2</sub>, N<sub>2</sub>, A, CO<sub>2</sub>, H<sub>2</sub>O), but is capable of ionizing nitric-oxide gas, presumably present as a trace constituent of the atmosphere in *D* region. There is reason to doubt that the increased ionization produced in *D* region during a solar flare is caused by a simple increase in solar Lyman  $\alpha$  emission [see 1 of "References" at end of paper], however, both because of the unreasonably large increases required to account for the observed lowering of the base of the *D* layer and because of the imperfect correlation between the time duration of observed H $\alpha$  emission and the duration of anomalous *D*-layer ionization. In addition, it has been known for several years that the sun normally produces a flux of soft X-rays, sufficiently intense to account for most of *E*-region ionization. This X-ray flux has been shown to vary considerably in magnitude and character in accordance with ground measurements of coronal excitation spectra [2]. Since an extension of the sun's normal X-ray emission spectrum to slightly shorter wavelengths could produce the increase in *D*-layer ionization observed during a flare, an X-ray theory of the origin of SID appeared to be an attractive alternative to the Lyman  $\alpha$  theory. The present paper describes a direct measurement made last summer of both the Lyman  $\alpha$  flux and the short wavelength X-ray flux reaching *D* layer during a solar flare that could be described as a large subflare or a flare of barely Class 1 magnitude.

## OPERATIONAL PROCEDURE

In order to determine whether an increase of Lyman  $\alpha$  radiation or X-ray radiation from the sun was responsible for the increase in *D*-layer ionization during a flare, it was planned to fly rocket-borne instrumentation to 100-km altitude in conjunction with ground measurements of flare activity. Since the scheduling and safety requirements of available land-based rocket ranges were likely to cause serious delays in attempts to launch rockets at times of flares, it was decided to carry out the measurements using balloon-borne six-inch solid-propellant rockets (Rockoons) fired at sea [3]. With this technique, it was possible to maintain a rocket suspended below a balloon in readiness for firing for a period of several hours. The operation was carried out from the U.S.S. *Colonial* (LSD-18), approximately 350 miles southwest of San Diego. The operational procedure was to launch a balloon and rocket in the morning. Once the balloon reached an altitude of 70,000 feet, the sun was observed continuously for evidence of a solar flare. The rocket could be command fired at floating altitude by radio. If no flare was observed by the time the balloon drifted to the boundary of the assigned firing area, the rocket had to be fired. During the operation, one rocket was successfully fired during a solar flare and two additional rockets gave background data on the level of solar Lyman  $\alpha$  emission at the present stage of the solar cycle.

## SUPPORTING MEASUREMENTS OF SOLAR FLARES

In order to determine when to launch the balloon-borne rockets, it was necessary to monitor concurrently the flare activity of the sun. Three methods of flare monitoring were attempted. Flares producing significant radio fadeout in the 6 to 15 Mc region were detected by monitoring the signal strength of four short-wave radio stations. The stations were so chosen that their signals were received via an ionospheric reflection occurring at a point on the sunlit side of the earth. During the operating period of the expedition, two such fadeouts were observed, but, unfortunately, they occurred on the one day during which no rocket launchings had been planned.

The detection of solar flares by means of radio fadeout is limited, in that it is capable of detecting only the infrequent larger flares producing intense low-level *D*-layer ionization. Since it appeared that the probability of encountering a large flare was small, direct observation of the sun was essential. We were fortunate in obtaining the cooperation of two solar observatories. Direct radio communication was established between the U.S.S. *Colonial* and the solar observatory at Sacramento Peak, Sunspot, New Mexico. Radio communication between the solar observatory at Climax, Colorado, and Sacramento Peak also permitted observations at Climax to be relayed to the operating ship. At Sacramento Peak, the solar disk was observed through a narrow band pass Lyot filter tuned to  $H\alpha$ . Measurements at Climax were made by means of a spectrohelioscope. Visual measurements were also attempted from shipboard by means of a narrow band filter and telescope assembly [4], mounted on a pointing control and utilizing television monitoring of the solar Ca K line emission. Constant cloud cover prevented visual observation of the sun from shipboard, and intermittent clouding also hampered observations from Sacramento Peak and Climax. The one flare during which a rocket was suc-

cessfully launched was observed at Climax. Without the generous cooperation of the people at both Climax and Sacramento Peak, these results could not have been obtained.

#### ROCKET INSTRUMENTATION

Four types of radiation detectors were flown in each of the rockets. An ion chamber with a LiF window and filled with nitric-oxide gas was used to measure the incident intensity of solar Lyman  $\alpha$  radiation; a Geiger counter with a 14.7 mg/cm<sup>2</sup> Be window was used to measure X-rays of wavelength less than 8Å; a partially shielded scintillation counter, using a NaI (TI) crystal 1-1/8 inches in diameter and 1/2 inch thick, combined with a 6199 photomultiplier, was flown as an X-ray pulse amplitude spectrometer to measure any hard X-ray flash that might be encountered during a flare. Two photocells sensitive to visible light were flown to determine rocket orientation in space [5]. One photocell measured the angle between the direction of the sun and the view plane of the detectors during each roll of the spinning rocket; the second photocell determined the phasing between horizon sweeps and views of the sun. The detector outputs were continuously recorded on the ground by means of a conventional four-channel FM, FM telemetering link with the rocket. The telemetering system was calibrated throughout the flight by use of a commutator in the rocket, which placed a 0 volt and a 2.64 volt signal on each channel input once every six seconds. The time of rocket firing was identified with signals from WWV.

The instrumentation in several of the rockets encountered varying problems of sudden failure at the time of rocket ignition. The failures affected individual information channels only. The channels that remained operating after rocket firing were in no way compromised by failures in other channels.

#### DESCRIPTION OF THE FLARE DURING WHICH ROCKET OBSERVATIONS WERE MADE

Rocket measurements were made during a flare reported to have begun between 19:05 and 19:07 UT, on July 20, 1956. This flare was observed on a spectrohelioscope by R. Hansen, at the High Altitude Observatory, Climax, Colorado. The information that the flare was in progress was immediately transmitted to our ship by radio via Sacramento Peak. At 19:15 UT, the H $\alpha$  from the affected region was still considerably brighter than preflare H $\alpha$ , but its strength was declining. At 19:20, there were still signs of a flare in progress, and shortly afterwards all evidence of a flare had disappeared. The flare was too small to produce a detectable radio fadeout. Unfortunately, no record of the flare is known to have been recorded at other solar observatories. The flare was considered by the High Altitude Observatory as being something between Class 1 and a subflare.

The rocket measurements were made between 19:16 UT and 19:18 UT. The rocket measurements did not, therefore, cover any initial flash excitation that may have occurred during this short-lived solar event. However, evidence of unusually intense coronal excitation was obtained.

#### EXPERIMENTAL RESULTS

The rocket measurements made during the flare of July 20 are shown in Figure 1. Curve A shows the current passed by the Lyman  $\alpha$  ion chamber as a function

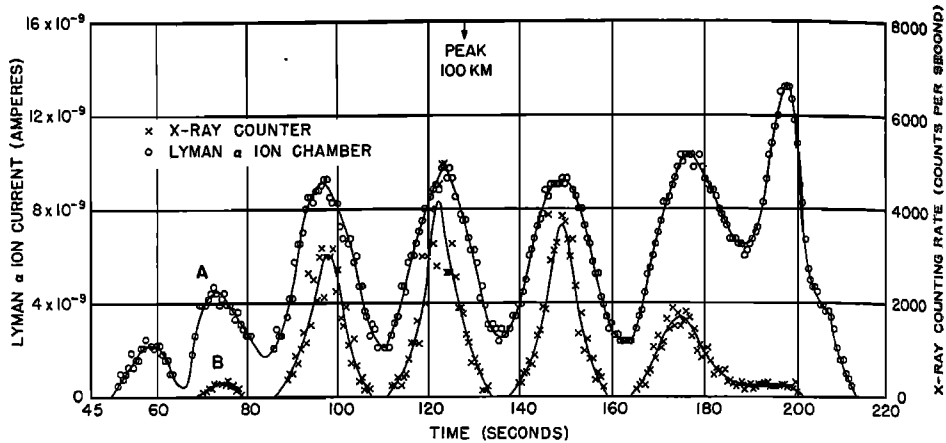


FIG. 1—Responses obtained from a nitric-oxide ion chamber and a beryllium window X-ray counter late in a Class 1 to 1— solar flare. The response of the X-ray counter has been corrected for circuit time constant and counter dead-time.

of flight time. The plotted points are the peak currents in the ion chamber corresponding to each complete roll of the rocket. This current is affected by the angle between the view plane and the sun. The current peaks observed at 97 seconds, 123 seconds, and 149 seconds were caused by the precession of the rocket, which at these altitudes behaved almost as a free-falling rigid body spinning mainly about its smallest moment of inertia. The rocket roll rate was 5.0 revolutions per second. The Lyman  $\alpha$  ion currents have been corrected only for a very slight non-linearity in the frequency shift of the FM modulator.

Curve *B* of Figure 1 shows the count rate response of the Be window Geiger counter. There is a considerable statistical variation in the individual response peaks of the Geiger counter, caused by the relatively small number of counts contributing to each counter response. The magnitude of the statistical fluctuations is attributable to the relatively brief view of the sun obtained during each roll. Each data point shown for Curve *B* is based on the average amplitude of three adjacent response peaks. The data have been corrected for the small non-linearity of the FM modulator, for circuit time constant (20 per cent), and for counter dead-time (450  $\mu$ -seconds).

The photomultiplier and aspect photocell information channels failed on rocket ignition.

#### LYMAN $\alpha$ RADIATION DURING THE FLARE

The determination of the intensity of Lyman  $\alpha$  radiation as measured by the "flare" rocket was complicated by lack of aspect photocell information. Despite this lack of direct aspect information, it proved possible to determine the incident Lyman  $\alpha$  intensity with fair accuracy. The manner by which the data were treated is described in the Appendix. Based on this treatment, the intensity of the above atmosphere flux of Lyman  $\alpha$  radiation was 6.1 ergs/cm<sup>2</sup> sec.

Table 1 shows a comparison between the incident Lyman  $\alpha$  flux measured in the flare rocket and the fluxes measured in other rockets utilizing the nitric-oxide

ion-chamber technique. The figure of  $\pm 20$  per cent reflects the confidence of the authors in the validity of the trajectory solution described in the Appendix.

TABLE 1—Solar Lyman  $\alpha$  emission as measured by nitric-oxide ion chambers

Rocket	Time of firing (UT)	Sun condition	Incident flux of Lyman $\alpha$
			<i>ergs/cm<sup>2</sup> sec</i>
A-34	2250, 10/18/55	Quiet	5.7 (-1 + 3)
A-35	0015, 10/22/55	Quiet	4.0 ( $\pm 0.8$ )
A-36	1530, 11/4/55	Quiet	9.2 ( $\pm 3$ )
D-5	1915, 7/17/56	Quiet	6.1 ( $\pm 0.3$ )
D-8	1917, 7/20/56	Late in Class 1 flare	6.1 ( $\pm 1.4$ )
D-13	2113, 7/25/56	Quiet	6.7 ( $\pm 0.3$ )

Clearly, there was no evidence of an unusually high value of Lyman  $\alpha$  radiation during the flare flight. It is not known whether the result indicates that small flares of the type encountered cause only a negligible percentage change in the over-all Lyman  $\alpha$  emission from the solar disk, or whether the Lyman  $\alpha$  emission accompanying the flare had decayed to negligible levels by the time the rocket measurements began. A comparison between the amplitudes of the Lyman  $\alpha$  maxima accompanying free-fall precession of the rocket, namely, those shown in Curve A of Figure 1 at 97 seconds, 123 seconds, and 149 seconds, shows that no major change in Lyman  $\alpha$  emission occurred during the flight.

SOLAR X-RAY EMISSION DURING THE SOLAR FLARE

In order to determine the intensity and to analyze the characteristics of the incident flux of solar X-ray radiation observed during the flare flight, it was necessary to correct the observed responses of Curve B, Figure 1, for variations in angle between the sun and the plane of view. The method of making this aspect correction is described in the Appendix. The large angle between the direction of the sun and the detector view plane which occurred during most of the flight resulted in a shift of the spectral sensitivity curve of the counter, as shown in Figure 2. The X-ray counting rates corrected for aspect are shown in Figure 3.

The X-ray intensities found in the flare flight are indicative of an exceptionally high state of coronal excitation. A comparison between the X-ray responses obtained during this flight and all previous flights for which we have X-ray data is shown in Table 2. The flare rocket received a 3A to 8A X-ray flux several times as intense as any flux previously measured. The high X-ray emission intensities found during the V-2 #49 flight may have been influenced by a Class 1 solar flare, which peaked 160 minutes before the flight. The relatively high energy value indicated for flight A-9 is partially the result of using a computation based on a  $2 \times 10^6$  degree K emission distribution. A computation based on a higher temperature would be justified for a Be window as thick as 47 mg/cm<sup>2</sup>. In contrast, the intensities observed since January 1953, flights A-14, A-16, and A-34, show that very low values of coronal emission below 8A occur during the sunspot minimum.

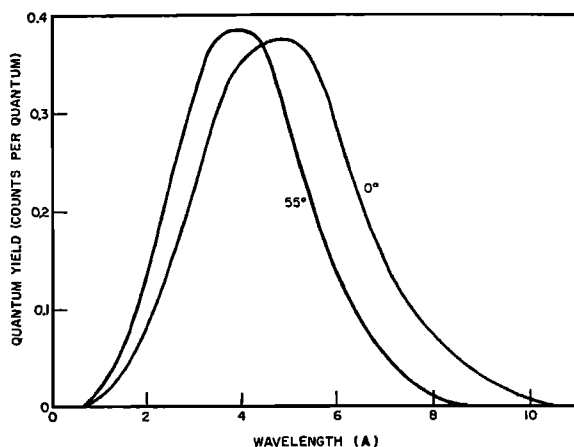


FIG. 2—Spectral response characteristics of 14.7 mg/cm<sup>2</sup> Be X-ray counter flown in the flare rocket. The spectral response is calculated from window and gas absorption coefficients for normally incident X-rays and for X-rays incident at an angle of 55°.

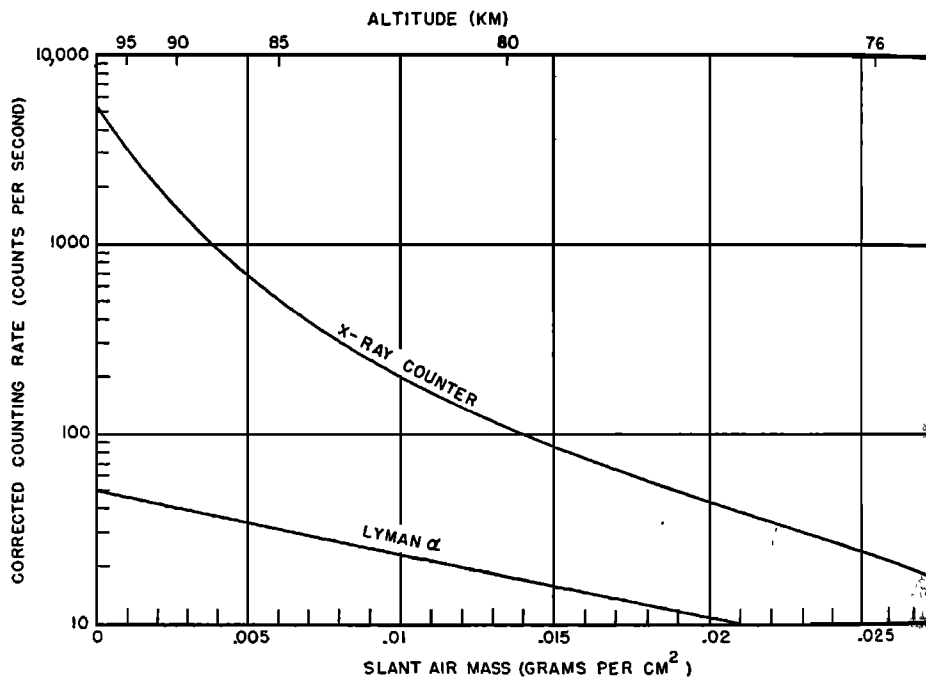


FIG. 3—Counting rate vs slant air mass for the Be window X-ray counter flown in the flare rocket. The counting rates have been corrected for changes in the angle between the detector view plane and the sun.

The A-14, A-16, and A-34 data are believed to be more reliable than those obtained previously.

#### ENERGY DISTRIBUTION OF FLARE

From the X-ray counting rate vs air mass curve of Figure 3, it has been possible to determine a smoothed energy spectrum of X-ray intensities. Such a spectrum

TABLE 2—Solar X-ray emission as measured by beryllium and aluminum Geiger counters

Rockets	Time of firing (UT)	Sun condition	Counterwindow material and surface density	Be counters response above atmosphere	Al counters response above atmosphere	Energy** below 8A	Energy** 8A to 20A
			mg/cm <sup>2</sup>	counts/cm <sup>2</sup> sec	counts/cm <sup>2</sup> sec	erg/cm <sup>2</sup> sec	erg/cm <sup>2</sup> sec
V-2 #49	1730, 9/29/49	160 min after Class 1 flare	Be 13	1.0 × 10 <sup>4</sup>	.....	0.0015	.....
A-9	1459, 5/1/52	Quiet	Be 47	495	.....	0.0017	.....
A-10	1344, 5/5/52	Quiet	Be 47	< 125	.....	< 0.0005	.....
Viking9	2138, 12/15/52	Quiet	Be 13	< 1.5 × 10 <sup>4</sup> **	.....	< 0.0006	.....
			Al 1.59	.....	2.9 × 10 <sup>4</sup> **	.....	0.2
A-14	2240, 11/15/53	Quiet	Be 13	< 40	.....	< 6.7 × 10 <sup>-4</sup>	.....
			Al 1.59	.....	< 3.1 × 10 <sup>4</sup> **	.....	< 0.0015
A-15	1546, 11/25/53	Quiet	Be 13	332	.....	2.9 × 10 <sup>-4</sup>	.....
			Al 1.59	.....	< 2.6 × 10 <sup>4</sup> **	.....	< 0.0013
A-16	1529, 12/1/53	Quiet	Al 1.59	.....	4.5 × 10 <sup>4</sup>	.....	0.0004
A-34	2250, 10/18/55	Quiet	Al 1.59	.....	1.4 × 10 <sup>5</sup>	.....	0.0012
D-8	1915, 7/20/56	Late in Class 1 flare	Be 14.7 at 55° aspect	1.2 × 10 <sup>5</sup>	.....	0.005	.....

\*These counters were filled with He + quench agent. Other counters were filled with Ne + quench agent. The Be window counters using He have about 1/20 the sensitivity of the Ne counters. The Al window counters using He have about 1/5 the sensitivity of the Ne counters.

\*\*A2 × 10<sup>4</sup> deg K gray-body emission curve is assumed for the sun. The spectrum is normalized to give the experimentally observed counting rates. The Table shows the energy of the normalized emission curve that falls within the indicated spectral limits.

can be calculated because the air absorption coefficients of the incident X-rays form a single valued and rapidly varying function of wavelength. Thus, we can subdivide the spectral region 3A to 9A into small spectral bands, each band having a unique range of air absorption coefficient. By approximating the curve of Figure 3 by a series of straight-line segments, each corresponding to a single absorption coefficient, we can determine the number of counts contributed by quanta in each spectral band. From the spectral sensitivity curves of Figure 2, the energy distribution of the incident flux can then be determined.

The results of such an analysis are shown in Figure 4. The energy spectrum determined in the above manner is not capable of showing any structure in the coronal emission characteristics, but does distinguish between thermal and non-thermal emission sources. For example, it is possible to distinguish a constant-emissivity (gray-body) single-temperature thermal emission from the non-thermal *Bremsstrahlung* produced by a monoenergetic electron beam injected into a cold gas. Similarly, a non-thermal synchrotron emission, analogous to that studied by C. Y. Fan, as a source of cosmic radio noise [6], is easily distinguished from the thermal emissions. On the other hand, it is not possible to differentiate between gray-body thermal emission and the *Bremsstrahlung* emission of a high-temperature electron cloud in a cold gas. Nor is it possible to differentiate between a constant emissivity source and one in which rapid fluctuations in emission occur with wavelength, so long as no single emission line overwhelmingly predominates in the spectral region covered.

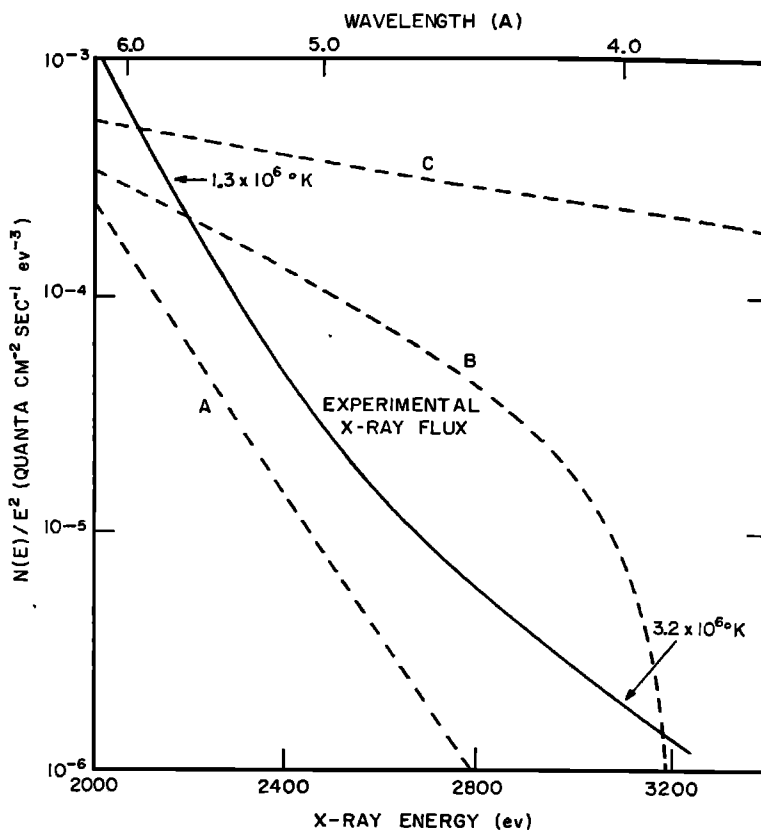


Fig. 4—Spectral distribution of incident X-ray quanta as a function of quantum energy. The plot of  $\log E^{-2}N(E)$  vs  $E$ , where  $N(E)$  is the number of quanta per  $\text{cm}^2$  per sec per unit energy interval, gives a straight line for gray-body emission for  $\lambda T < 0.3$ . The heavy solid line is a plot of the experimental energy distribution observed during the solar flare. The three broken-line curves show the relative emission spectrum from (A) a  $2 \times 10^6$  degree K electron cloud in a cool gas, (B) a stream of 3,200 eV monoenergetic electrons injected into a cool gas, and (C) a high energy  $E^{-2}$  distribution of electrons radiating in synchrotron fashion due to acceleration in a magnetic field.

The observed emission spectrum of the coronal X-rays most closely approximates the thermal type with the form to be expected from a collection of solar regions at different states of temperature excitation. If the results are interpreted in terms of emission from gas of approximately constant emissivity, emission from regions of  $T = 1.3 \times 10^6$  degrees K predominated at 6A, and emission at  $T = 3.2 \times 10^6$  degrees K predominated at 4A. If the results are interpreted in terms of *Bremsstrahlung* emission from a highly excited electron gas in a much colder scattering medium, emission from regions of electron temperature  $T_e = 1.6 \times 10^6$  degrees K predominated at 6A, and emission from regions of  $T_e = 4.41 \times 10^6$  degrees K predominated at 4A. This general form of coronal emission is in agreement with the interpretation of data obtained at longer wavelengths in Aerobee 16.

#### CONCLUSIONS

On the basis of measurements of 3A to 8A X-rays and Lyman  $\alpha$  radiation made



from a rocket flown late in a Class 1 to Class 1— solar flare, there was observed an unusually large amount of X-ray emission below 8A. The character of the emission in the 3A to 8A region resembles a collection of thermal sources at temperatures ranging from  $1.0 \times 10^6$  degrees C to  $3.5 \times 10^6$  degrees C. The intensity of Lyman  $\alpha$  radiation during the flight was comparable to that observed from other rockets fired in the same two-week period. In view of the fact that the visible emission of H $\alpha$  from the flare region had almost returned to normal by the time of the rocket measurements, the emissions observed are indicative of a strong intensification of the normally present coronal excitation. The measurements would seem to indicate that the coronal excitation lingers long after the primary flare event has dissipated. They also lend strong support to the theory that a hardening of the coronal X-ray spectrum is the source of the low-level D-layer ionization responsible for SID.

It is planned to fire additional rockets coincident with direct observations of solar flares in the summer of 1957. The development of an island-based launching facility will permit ground launchings. With this arrangement, attempts will be made to obtain measurements during various phases of larger SID producing flares.

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### APPENDIX

#### *Treatment of Data*

The basic data upon which the incident intensities of solar X-ray and Lyman  $\alpha$  radiation were determined are shown in Figure 1. In order to reduce these data to absolute energy values, it was necessary to determine the angle between the detector view plane and the sun. The solution of the problem was greatly facilitated by knowledge of the typical behavior of the type of rocket flown, as determined from other flights during which photocell aspect information was obtained. It was also facilitated by the relatively wide angular response of the ion chamber, which made the response relatively insensitive to small changes in aspect when the aspect was good.

The key to the reduction of the Lyman  $\alpha$  data for the flare rocket, Curve A, Figure 1, lies in the Lyman  $\alpha$  response peak observed at 197.5 seconds and the shoulder observed at 207 seconds, combined with a knowledge that the other Deacon rockets of comparable spin rate tumbled due to air drag at altitudes above the altitude at which solar Lyman  $\alpha$  radiation disappeared. As the rocket tumbles and subsequently swings, it must pass through perfect aspect at least

once. The solution of the rocket aspect problem in this case is based on the assumption that the rocket had perfect aspect both at the 197.5 second peak and at the 207 second shoulder. The difference in the amplitudes of the two response peaks must then be due to atmospheric attenuation. Since the rocket peaked within a few seconds of 128 seconds, its vertical velocity in the neighborhood of 200 seconds was known fairly accurately. Thus, from a knowledge of the scale height in  $D$  region, the atmospheric attenuation at 197.5 seconds was determined. The atmospheric attenuation at 197.5 seconds determined in this manner was 0.51. The corresponding above atmosphere incident flux of Lyman  $\alpha$  radiation was 6.1 ergs/cm<sup>2</sup> sec. The trajectory determined by this solution gives a peak altitude of 100.1 km and a peak time of 128 seconds. This trajectory was very similar to that of another Deacon flight in the same series, which had a peak altitude of 102.5 km and a peak time of 128 seconds, as determined from matching aspect corrected ascent and descent Lyman  $\alpha$  data in conjunction with Rocket Panel [7] pressure data. An independent check of the accuracy of the flare rocket trajectory solution can be gained from a comparison in the computed altitudes of the last observed Lyman  $\alpha$  responses for these two rockets. Since the rockets are both tumbling rapidly at these altitudes, differences in aspect are unlikely to have major effects on the results. The last response of the flare rocket was noted at 212 seconds, corresponding to 66.6 km. The last response of the comparison flight occurred at 215 seconds, corresponding also to 66.6 km.

In order to reduce the X-ray data, it was necessary to correct the observed X-ray responses, Curve *B*, Figure 1, for the continually varying angle between the detector view plane and the sun. This aspect correction was made by using the response of the Lyman  $\alpha$  ion chamber, corrected for atmospheric attenuation as a measure of the rocket aspect. The relative response of this type of ion chamber as a function of the angle between its direction of view and the sun was determined from the responses obtained in flights with operating photocells. By using this angular response curve, it was determined that at the aspect peak of 123 seconds, the sun was 55° above the plane of view swept out by the detectors. All data obtained at times such that the aspect angle was greater than 55° were adjusted on the basis of an aspect correction curve obtained by plotting Lyman  $\alpha$  ion-chamber response *vs* counter response for points obtained within 51 seconds of peak time. These data were obtained at almost constant altitude, that is, within 1.0 km of peak. This method of correction required no knowledge of either the angular response curve of the ion chamber or of the counter. All data obtained at times such that the angle between sun and detector view plane was less than 55° were corrected on the basis of the ion-chamber angular-response curve deduced from other flights and from an X-ray angular-response curve calculated from the counter geometry. Calculations showed that the counter angular response was determined mainly by the free window area projected on a plane perpendicular to the line between detector and sun. In addition, there was a shift in spectral response due to increased effective window thickness and an increased path length through the absorbing counter gas. This shift in spectral response is shown in Figure 2. The X-ray counting rates plotted in Figure 3 have been corrected in this manner to a constant aspect angle of 55°.