TECHNICAL NOTE

Polarizing properties of grazing-incidence x-ray mirrors: comment

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We show that grazing-incidence telescopes, like those used for x-ray imaging, present negligible instrumental polarization. This property does not depend on the number of reflections the telescope employs to lead light from the entrance pupil to the focal plane. The result applies to the various mirrors of the advanced x-ray astrophysics facility satellite. In this particular case we have quantified the residual instrumental polarization to be between 10^{-3} and 5×10^{-5}, depending on the size of the resolution elements.

Key words: Polarimetry, instrumental polarization, x-ray telescopes.

1. Introduction

In a recent paper Chipman *et al.*[1] analyzed the instrumental polarization in the focal plane of the high-resolution mirror assembly (HRMA), i.e., the image-forming system of the advanced x-ray astrophysics facility (AXAF).[2] In particular in their study they attempt to decide whether polarimetric measurements in the focal plane of this x-ray telescope are possible. They conclude that polarimetry is feasible, but, according to them, it is possible because of a fortunate accident in the design of the mirrors. ("Had the HRMA been designed with an odd number of mirrors polarimetry would be infeasible with the telescope.") In order to be led from the entrance to the focal plane, each incoming ray suffers two grazing-incidence reflections. According to Chipman and co-workers, each one creates a huge instrumental polarization, although both together compensate to produce a negligible net effect. Consequently these authors find and discuss a set of striking properties of a particular type of grazing-incidence telescope, i.e., those instruments with an odd number of reflections. For example, they claim the following:

(1) "The primary mirror acts as a spatial depolarizer by scrambling linear polarization states and making polarimetry impossible at the focal plane of the primary mirror."

(2) "The center of the point spread function is dark, not a maximum as is usual."

To model possible uncertainties in the polarimetry performed with the large earth-based solar telescope (LEST),[3] we developed a theory to compute the diffraction pattern in the focal plane of a telescope when the input light is partly polarized.[4] Our technique is also suitable for modeling the mirrors of AXAF, but it does not reproduce the results quoted above. After a thorough study we found the source of disagreement between the study of Chipman *et al.* and our method. We concluded that Chipman and co-workers overestimated by a large amount the change in polarization produced after a grazing-incidence reflection. Consequently, the predictions given in their paper and quoted above are basically incorrect. We claim that grazing-incidence telescopes do not seriously limit the polarimetry one can perform with them, independently of the number of reflections they might use. In other words the polarization of the light when it reaches the focal plane of one of these telescopes remains almost as it was at the entrance of the telescope.

The goal of the present study is to describe the predictions of our model as well as to quantify the instrumental polarization of the individual mirrors of HRMA. With this aim in mind, in Section 2 we discuss the change of polarization suffered by a plane wave after a grazing reflection. Section 3 is devoted to the equations that describe the polarizing properties of a grazing-incidence, image-forming mirror. Finally in Section 4 we discuss the instrumental polarization of HRMA. We also propose simple em-

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pirical tests that will access the reliability of our predictions.

2. Polarizing Properties of a Grazing-Incidence Reflection

Grazing-incidence reflections keep the polarization of the reflected beams almost unchanged. If one characterizes the polarization of the light by using the complex amplitude of its electric field (the so-called Jones parameters[5]), the variations in polarization that occur in a reflection are described by the Fresnel coefficients.[5,6] These two coefficients give the ratios between the Jones parameters of the reflected and the incident beams. When the coordinate systems in Fig. 1 are used, these Fresnel coefficients turn out to be[5,6]

$$a_p = |a_p|\exp(j\delta_p) = [n^2 \cos\phi - (n^2 - \sin^2\phi)^{1/2}]/[n^2\cos\phi + (n^2 - \sin^2\phi)^{1/2}],$$

$$a_s = |a_s|\exp(j\delta_s) = [\cos\phi - (n^2 - \sin^2\phi)^{1/2}]/[\cos\phi + (n^2 - \sin^2\phi)^{1/2}], \quad (1)$$

where n represents the complex refractive index of the reflective surface, ϕ stands for the incidence angle, and j symbolizes the imaginary unit $(-1)^{1/2}$. The coefficients a_p and a_s provide the ratios for light vibrating in the incidence plane and in the perpendicular plane, respectively (see Fig. 1). Their moduli ($|a_p|$, $|a_s|$) and arguments (δ_p, δ_s) have been written down explicitly in Eq. (1). Grazing incidence implies that $\phi \simeq 90°$ so that $\sin\phi \simeq 1$ and $\cos\phi \simeq 0$. Therefore the Fresnel coefficients in Eq. (1) become in this extreme case

$$a_s \simeq a_p \simeq -1. \quad (2)$$

Such a property does not depend on the type of material covering the surface, and its meaning is quite clear: because $a_s \simeq a_p$, the reflected light remains with the Jones parameters of the incident light (except for an irrelevant global factor). The values for the Fresnel coefficients depend on the coordinate systems used to express the electric fields of the incident and reflected lights (for example, a change in the direction of one of the axes artificially varies by 180° the argument of one of the Fresnel coefficients). Nevertheless, according to the criterion adopted in Fig. 1, the input and output reference systems become the same when ϕ approaches 90°. This fact renders the interpretation of relation (2) straightforward; i.e., grazing-incidence reflections keep the input polarization unchanged. As an example of how a_s and a_p tend to behave as described by relation (2), we show in Fig. 2 the values of their moduli and phases as a function of the incidence angle. We adopt a refractive index $n = 0.91 - j0.04$ because it is representative of the optical properties of the AXAF at x-ray wavelengths.[1] We have also included in the same figure the modulus and phase of the ratio a_p/a_s, which approaches unit for $\phi \sim 90°$. The source of error in the treatment by Chipman *et al.*[1] starts at this point. They mistakenly used $a_s/a_p \sim -1$ when $\phi \sim 90°$. In other words, they considered that a grazing-incidence reflection resembles a half-wave plate. This misleading minus sign causes the large effects on the instrumental polarization that, according to their claim, grazing-incidence,

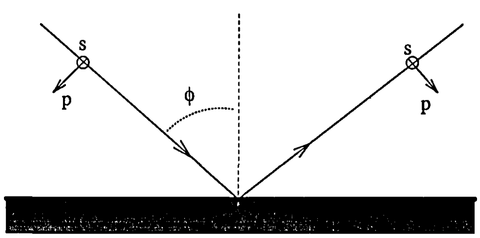

Fig. 1. Input and output reference systems used to write down the Fresnel coefficients corresponding to a reflection. The terms p and s are the unit vectors parallel and perpendicular, respectively, to the plane of incidence; ϕ stands for the angle of incidence, i.e., the angle between the incident ray and the normal to the reflective surface (represented by the dark strip).

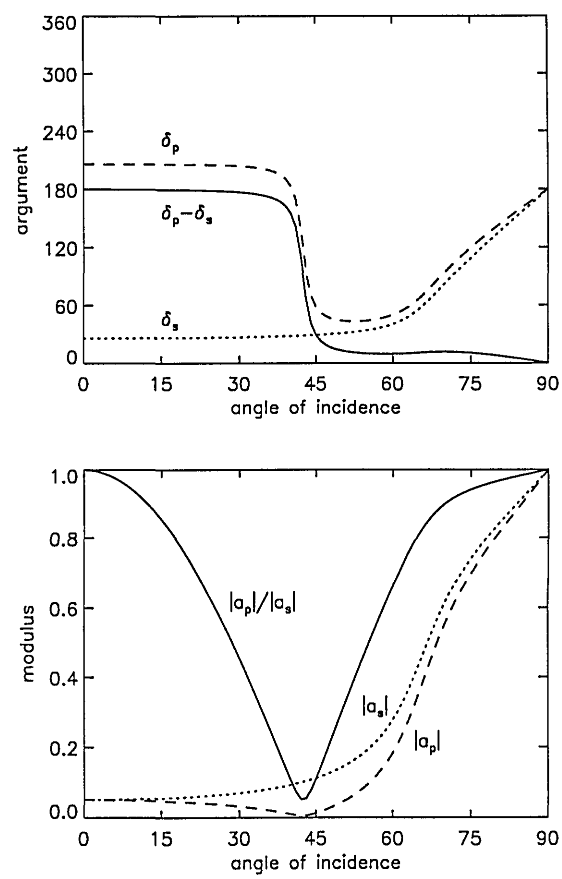

Fig. 2. Moduli ($|a_p|$, $|a_s|$) and arguments (δ_p, δ_s) of the Fresnel coefficients a_p and a_s as a function of the incidence angle (see text for details). The modulus and argument of the ratio a_p/a_s are also included in the figures. It is evident that grazing incidence (incidence angle, ~90°) implies $a_p/a_s \sim 1$ (i.e., $\delta_p - \delta_s \sim 0$ and $|a_p|/|a_s| \sim 1$). Arguments and angles are given in degrees.

single-reflection telescopes should present. We have introduced the expression single-reflection telescope to denote all those telescopes that employ a single reflection to lead each input ray to its focal plane.

3. Instrumental Polarization of an Ideal Single-Reflection Grazing-Incidence Telescope

In keeping with our previous study,[4] the diffraction pattern produced close to the axis of an image-forming instrument was expressed by a matrix. This so-called Mueller matrix[7] relates the Stokes parameters of a plane wave entering the system to the Stokes parameters of the light reaching each single point of the focal plane. Our model considers that different rays of an incoming beam suffer different variations in their original polarization along their paths through the system. Diffraction effects are also included because the net change of polarization corresponds not to any of the individual rays but to the coherent average of the whole bundle of rays reaching the focal plane. In particular the Mueller matrix of an axisymmetric optical system, illuminated by a point source lying on its axis, turns out to be[4]

$$\begin{bmatrix} |h_+|^2 + |h_-|^2 & -2\cos(2\theta)\mathrm{Re}\{h_+h_-^*\} & -2\sin(2\theta)\mathrm{Re}\{h_+h_-^*\} & 0 \\ -2\cos(2\theta)\mathrm{Re}\{h_+h_-^*\} & |h_+|^2 + |h_-|^2\cos(4\theta) & |h_-|^2\sin(4\theta) & -2\sin(2\theta)\mathrm{Im}\{h_+h_-^*\} \\ -2\sin(2\theta)\mathrm{Re}\{h_+h_-^*\} & |h_-|^2\sin(4\theta) & |h_+|^2 - |h_-|^2\cos(4\theta) & 2\cos(2\theta)\mathrm{Im}\{h_+h_-^*\} \\ 0 & 2\sin(2\theta)\mathrm{Im}\{h_+h_-^*\} & -2\cos(2\theta)\mathrm{Im}\{h_+h_-^*\} & |h_+|^2 - |h_-|^2 \end{bmatrix}, \tag{3a}$$

where

$$h_+(q) = \int_0^1 [A_\rho(x) + A_\theta(x)] J_0(xq) x \mathrm{d}x,$$

$$h_-(q) = \int_0^1 [A_\rho(x) - A_\theta(x)] J_2(xq) x \mathrm{d}x,$$

$$q = k\rho R/f. \tag{3b}$$

The reference systems used to express the matrix are arbitrary except that the axes of the input and output systems have to be parallel.[4] A set of new symbols has been introduced together with these equations: k, R, and f represent the wave number, the radius of the entrance pupil, and the focal length of the system, respectively. The terms ρ and θ stand for the polar coordinates radial distance (relative to the optical axis) and azimuthal angle in the focal plane. Note that there is a matrix (3a) for each pair ρ and θ. J_1 symbolizes the Bessel function of first kind and order 1, while h_-^* is the complex conjugate of h_-. Finally the coefficients A_ρ and A_θ give, for each incident ray, the ratio between the Jones parameters at the entrance and at the focal plane. A_ρ and A_θ relate the components in the radial and azimuthal directions, respectively. Because of our assumption concerning axial symmetry, A_ρ and A_θ depend only on x, the distance to the axis of the optical system at the entrance pupil. We use a normalized distance x so that it reaches unity at the entrance pupil's radius. In the case of grazing-incidence mirrors, such as those we are dealing with, only rays within an annular aperture are focused. A_ρ and A_θ correspond to the Fresnel coefficients [Eqs. (1)] within the annulus, and they are zero elsewhere, namely,

$$A_\rho(x) = \begin{cases} 0 & 0 \le x \le \epsilon \\ a_p & \epsilon < x \le 1 \end{cases},$$

$$A_\theta(x) = \begin{cases} 0 & 0 \le x \le \epsilon \\ a_s & \epsilon < x \le 1 \end{cases}, \tag{4}$$

where ϵ stands for the central obscuration ratio (the ratio between the internal and external radii of the annular aperture).

A_ρ and A_θ should be identical throughout the entrance pupil of ideal imaging optical systems that do not alter the input polarization. Consequently, according to our matrix formalism, these nonpolarizing optical systems are characterized by having $|h_-| = 0$ [see Eqs. (3b)], the corresponding Mueller matrices (3a) being proportional to the identity matrix. Polarizing systems deviate from this matrix because of the coefficients $h_+h_-^*$ and $|h_-|^2$ [see once more (3a)]. Therefore these coefficients determine the importance of the polarizing properties of a given optical system.

We aim to set limits on the instrumental polarization produced by the individual mirrors of AXAF. Following our previous argumentation, we need to compute $h_+h_-^*$ and $|h_-|^2$ and afterward compare them with $|h_+|^2$. HRMA mirrors have a central obscuration ϵ as large as ~ 0.98.[1] When x in Eq. (3b) falls between 0.98 and 1.00, the incidence angle ϕ moves from 89.09 to 89.56 deg.[1] Taking into account that ϕ is almost constant and considering that we want only to estimate an order of magnitude, the coefficients a_p and a_s can be pulled out of the integral signs once Eq. (4) is subsituted into Eqs. (3b). This simplification allows us to obtain analytical expressions for both h_+ and h_- (Ref. 8):

$$h_+(q) \simeq (a_p + a_s) \int_\epsilon^1 \mathbf{J}_0(xq)x\mathrm{d}x$$

$$= (a_p + a_s)[\mathbf{J}_1(q) - \epsilon\mathbf{J}_1(\epsilon q)]/q,$$

$$h_-(q) \simeq (a_p - a_s) \int_\epsilon^1 \mathbf{J}_2(xq)x\mathrm{d}x$$

$$= (a_s - a_p)\{2[\mathbf{J}_0(q) - \mathbf{J}_0(\epsilon q)]/q^2 + [\mathbf{J}_1(q) - \epsilon\mathbf{J}_1(\epsilon q)]/q\}. \quad (5)$$

Using Eq. (1) and relations (5) with $\epsilon = 0.98$, $n = 0.91 - j0.04$, and $\phi = 89.09$ deg, we show in Fig. 3 the various parameters that characterize the Mueller matrix of the system. We plot, as a function of the normalized radial coordinate in the focal plane q, the coefficients $|h_+h_-^*|$ and $|h_-|^2$, which modulate the instrumental polarization of our annular mirror. Three orders of magnitude separate the maximum $|h_+|^2$ (which quantifies the nonpolarizing part of the system) from these coefficients. Therefore in practice input beams of arbitrary polarization will produce images that keep the original polarization and whose intensities are distributed according to $|h_+|^2$. This coefficient becomes a maximum in the optical axis so that the diffraction patterns will always present a bright central spot. Note that the nonpolarizing character of the grazing-incidence imaging mirrors does not depend on the actual value of the refractive index.

Depending on the size of our resolution elements, the matrix (3a) might not represent the relationship between input and measured polarizations. If one integrates in each pixel the whole diffraction pattern, the Mueller matrix that relates the input Stokes parameters to the measured Stokes parameters is the average of (3a) throughout the focal plane. Except

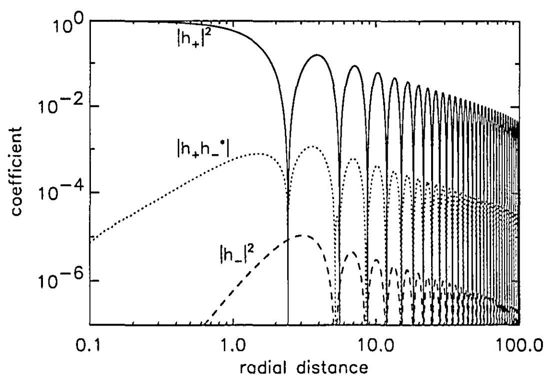

Fig. 3. Various coefficients that appear in the Mueller matrix of a single-reflection, grazing-incidence mirror versus normalized radial distance in the focal plane q (see text for details). The abscissa and ordinate are plotted with logarithmic scales.

for a global factor this mean Mueller matrix becomes[4]

$$\begin{bmatrix} H_+ + H_- & 0 & 0 & 0 \\ 0 & H_+ & 0 & 0 \\ 0 & 0 & H_+ & 0 \\ 0 & 0 & 0 & H_+ - H_- \end{bmatrix}, \quad (6a)$$

with

$$H_+ = \int_0^\infty |h_+(q)|^2 q\mathrm{d}q,$$

$$H_- = \int_0^\infty |h_-(q)|^2 q\mathrm{d}q. \quad (6b)$$

When the values for h_+ and h_- given in Eq. (5) are used, the integrals for H_+ and H_- can be solved analytically.[8,9] It turns out that

$$H_+ = |a_p + a_s|^2(1 - \epsilon^2)/2,$$

$$H_- = |a_p - a_s|^2(1 - \epsilon^2)/2, \quad (7)$$

or, by employing the same refractive index and incidence angle as in Fig. 3,

$$H_-/H_+ \simeq 4.7 \times 10^{-5}. \quad (8)$$

Accordingly, if the pixel sizes involved in our observations are large compared with the diffraction patterns, our imaging system is nonpolarizing with a precision of $\sim 5 \times 10^{-5}$. This figure corresponds to the relative deviation of Eq. (6a) from the identity matrix.

4. Comments and Conclusions

Our predictions are in disagreement with those of Chipman *et al.*,[1] which triggered the present paper. One can think of various tests to verify them. According to Chipman *et al.*,[1] the diffraction pattern of a single-reflection, grazing-incidence mirror should present a dark spot at its center.[10] This is not present in our model (see Fig. 3). A second and simple test can be realized with a mirror whose imaging properties are not necessarily limited by diffraction. According to Chipman *et al.*,[1] an input linearly polarized beam produces an image that, once averaged throughout the focal plane, is depolarized [see their Eq. (25)]. We predict [Eqs. (6a) and (8)] that the system will keep the linear input polarization. Both tests can be performed by using visible light since none of the theories seriously relies on the actual values of the refractive indices. They basically depend on the grazing-incidence character of the imaging mirrors.

Although the results in Section 3 correspond to an ideal single-reflection mirror limited by diffraction (i.e., free of aberration), we think that they provide the correct order of magnitide for the effects that polarization could induce in realistic x-ray telescopes. Differences between real telescopes and the model do not invalidate its predictions. For example, x-ray telescopes are not limited by diffraction. Nevertheless one can show that the average Mueller matrix (6a) is almost independent of possible aberrations.[4] On the other hand, actual telescopes increase their collecting surfaces, adding up in a common focal plane the photons gathered by various concentric annular mirrors. Yet again, the average Mueller matrix remains as in our ideal case because the addition of several matrices (6a) produces a matrix like the original (except for a constant factor). A third difference might be that the light in its path through the optical system suffers several reflections instead of just one (two in the particular case of the HRMA AXAF). The figures computed in Section 3 vary by less than 1 order of magnitude after this is taken into account.[11] Therefore, considering that the figures just give an order of magnitude for the instrumental effect, it is possible to say that x-ray telescopes present instrumental polarization between 10^{-3} and 5×10^{-5}. This varies with the size of the resolution element so that the larger the pixel the smaller the effect. Such instrumental polarizations seem negligible for any practical application.

In summary, grazing-incidence telescopes are extremely well suited for polarimetry. This property does not depend on the number of reflections used by the telescope to lead light from the entrance pupil to its focal plane.

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References and Notes

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9. I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products* (Academic, London, 1980).
10. Chipman *et al.*[1] quote a personal communication claiming that in fact this dark central spot has been observed. In view of the significance of this observation, it will be quite useful to publish such laboratory measurements confirming or denying the existence of a dark spot.
11. Assume that the rays in their paths through the system suffer n identical reflections. In this case the equations in Section 3 remain valid. Although they have been computed for a single-reflection mirror, the formalism still works if one substitutes a_p^n and a_s^n for a_p and a_s in Eq. (4). By following the evolution of this change through the relevant equations, it is easy to show that the polarizing terms of Fig. 3 increase by a factor of $\sim n$. Similarly, the polarizing terms of the average Mueller matrix (6a) vary as n^2. In the particular case of AXAF, $n = 2$, so our numerical simulations suffer a change smaller than 1 order of magnitude.