Using visibilities in the electron space: applications

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Solar Activity during the Onset of Solar Cycle 24
Napa, December 11th 2008

www.dima.unige.it/~piana/mida/group

Joint work

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Outline

- Flare location
- Example of application to simulated data
- Example of application to a real flare

Flare location

Centroid of the X-ray emission

$$x_c(\varepsilon) = \frac{\int_x \int_y xI(x, y; \varepsilon)dxdy}{\int_x \int_y I(x, y; \varepsilon)dxdy} \quad , \quad y_c(\varepsilon) = \frac{\int_x \int_y yI(x, y; \varepsilon)dxdy}{\int_x \int_y I(x, y; \varepsilon)dxdy}$$

Centroid of the electron distribution in the source

$$x_c(E) = \frac{\int_x \int_y xN(x, y)\overline{F}(x, y; E)dxdy}{\int_x \int_y N(x, y)\overline{F}(x, y; E)dxdy} \quad , \quad y_c(E) = \frac{\int_x \int_y yN(x, y)\overline{F}(x, y; E)dxdy}{\int_x \int_y N(x, y)\overline{F}(x, y; E)dxdy}$$

X-ray emission centroid

How to calculate the centroid of the X-ray emission from the X-ray visibilities provided by RHESSI?

apply forward fit with a general shape
(elliptical gaussian, two circular gaussians, loop)

or

make a map with back-projection or MEM and
use numerical integration to calculate the integrals

Electron distribution centroid

How to calculate the centroid of the electron distribution in the source from the X-ray visibilities provided by RHESSI?

1. build a set of electron visibilities

(Piana et al. 2007, *ApJ* **665**, 846)

2. apply forward fit with a general shape (elliptical gaussian, two circular gaussians, loop)

or

make a map with back-projection or MEM and use numerical integration to calculate the integrals

Centroids from visibilities: X-rays

X-ray visibilities: $$V(u,v;\varepsilon)=\int_x\int_y I(x,y;\varepsilon)e^{2\pi i(ux+vy)}dxdy$$

Derivatives:
$$\left\{\begin{aligned}\int_x\int_y xI(x,y;\varepsilon)dxdy &= \frac{1}{2\pi i}\frac{\partial V}{\partial u}(0,0;\varepsilon)\\ \int_x\int_y yI(x,y;\varepsilon)dxdy &= \frac{1}{2\pi i}\frac{\partial V}{\partial v}(0,0;\varepsilon)\end{aligned}\right.$$

Centroid:
$$\left\{\begin{aligned}x_c(\varepsilon) &= \frac{\int_x\int_y xI(x,y;\varepsilon)dxdy}{\int_x\int_y I(x,y;\varepsilon)dxdy}=\frac{1}{2\pi i}\frac{1}{V(0,0;\varepsilon)}\frac{\partial V}{\partial u}(0,0;\varepsilon)\\ y_c(\varepsilon) &= \frac{\int_x\int_y xI(x,y;\varepsilon)dxdy}{\int_x\int_y I(x,y;\varepsilon)dxdy}=\frac{1}{2\pi i}\frac{1}{V(0,0;\varepsilon)}\frac{\partial V}{\partial v}(0,0;\varepsilon)\end{aligned}\right.$$

From X-rays to electrons

X-ray visibilities:

$$V(u,v;\varepsilon) = \int_x \int_y I(x,y;\varepsilon) e^{2\pi i(ux+vy)} dxdy$$

Electron visibilities:

$$W(u,v;E) = \int_x \int_y N(x,y) \overline{F}(x,y;E) e^{2\pi i(ux+vy)} dxdy$$

Bremsstrahlung equations:

$$I(x,y;\varepsilon) = \int_\varepsilon^\infty N(x,y) \overline{F}(x,y;E) Q(\varepsilon,E) dE$$

$$V(u,v;\varepsilon) = \int_\varepsilon^\infty W(u,v;E) Q(\varepsilon,E) dE$$

Centroids from visibilities: electrons

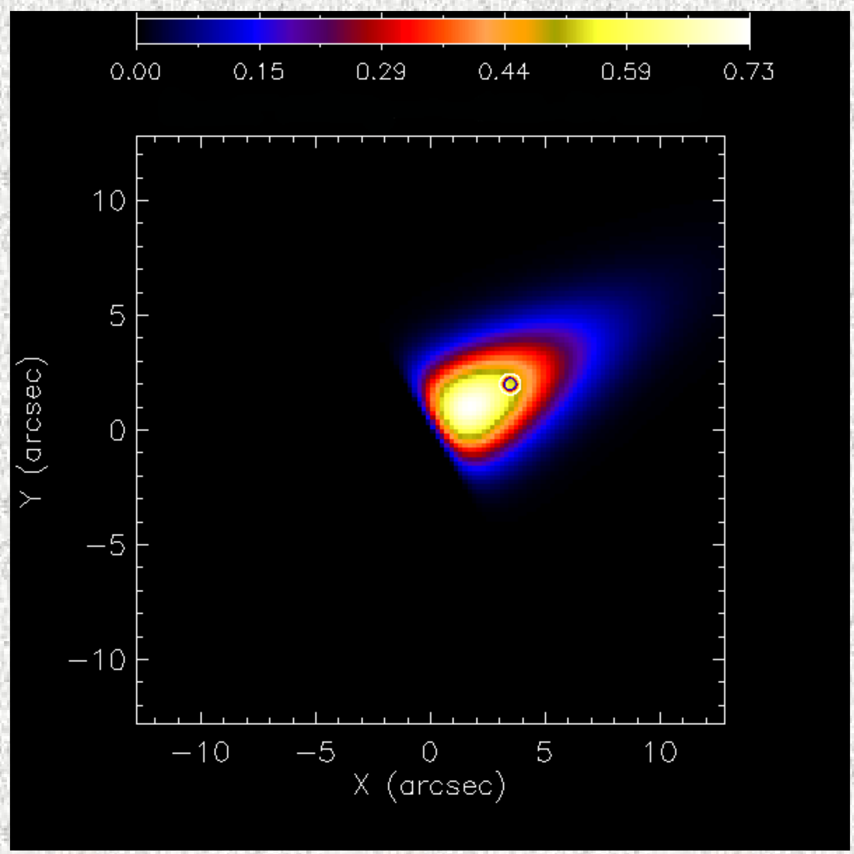
Electron visibilities: $W(u,v;E)=\int_x\int_y N(x,y)\overline{F}(x,y;E)e^{2\pi i(ux+vy)}dxdy$

Derivatives:
$$\begin{cases}\int_x\int_y xN(x,y)\overline{F}(x,y;E)dxdy=\dfrac{1}{2\pi i}\dfrac{\partial W}{\partial u}(0,0;E)\\ \int_x\int_y yN(x,y)\overline{F}(x,y;E)dxdy=\dfrac{1}{2\pi i}\dfrac{\partial W}{\partial v}(0,0;E)\end{cases}$$

Centroid:
$$\begin{cases}x_c(E)=\dfrac{\int_x\int_y xN(x,y)\overline{F}(x,y;E)dxdy}{\int_x\int_y N(x,y)\overline{F}(x,y;E)dxdy}=\dfrac{1}{2\pi i}\dfrac{1}{W(0,0;E)}\dfrac{\partial W}{\partial u}(0,0;E)\\ y_c(E)=\dfrac{\int_x\int_y yN(x,y)\overline{F}(x,y;E)dxdy}{\int_x\int_y N(x,y)\overline{F}(x,y;E)dxdy}=\dfrac{1}{2\pi i}\dfrac{1}{W(0,0;E)}\dfrac{\partial W}{\partial v}(0,0;E)\end{cases}$$

Example: simulation

Synthetic map



Skewed function

$$f(s) \approx s e^{-s}$$

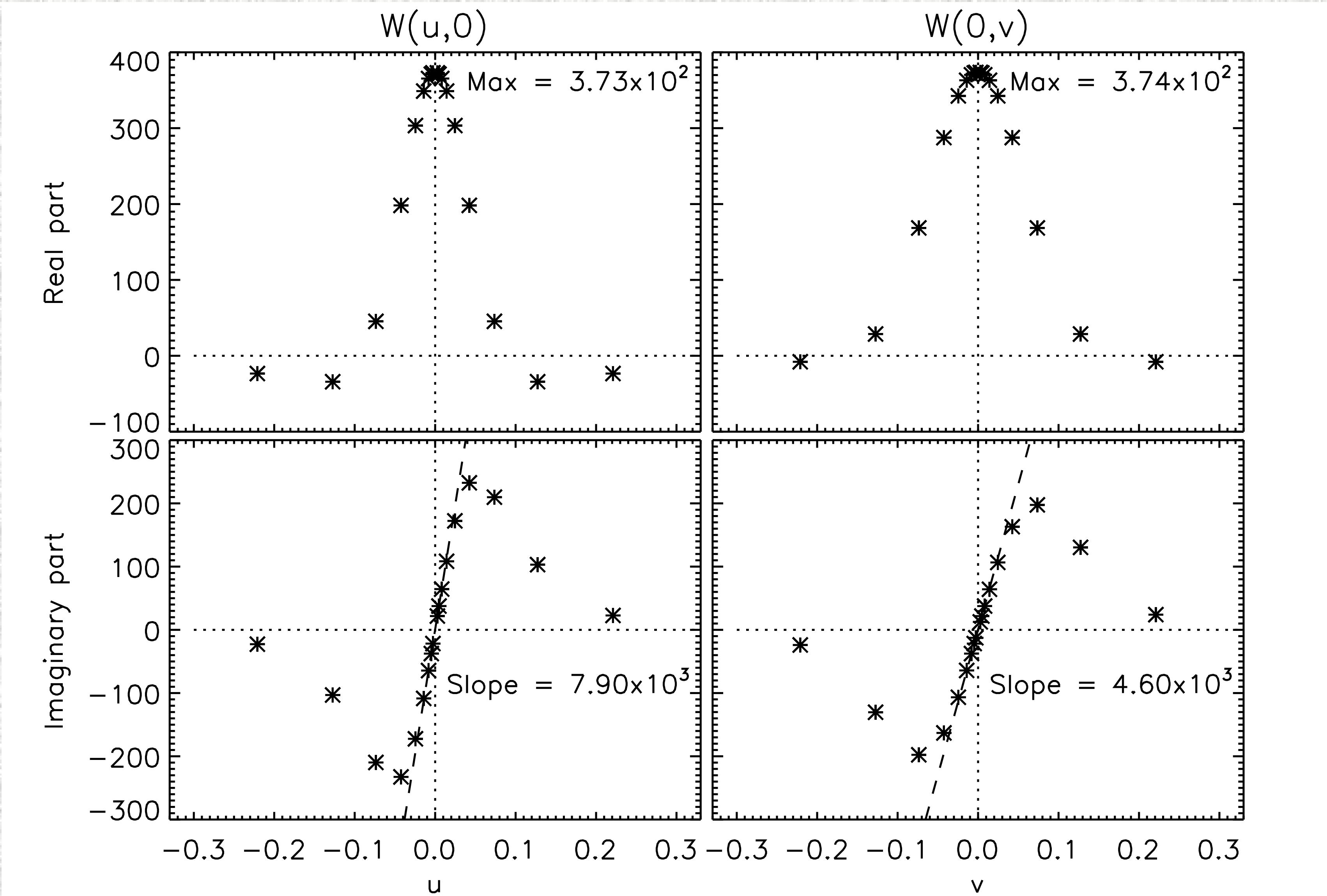
Centroid

$(x_c , y_c) = (3.464 , 2.000)$

Peak

$(x_p , y_p) = (1.732 , 1.000)$

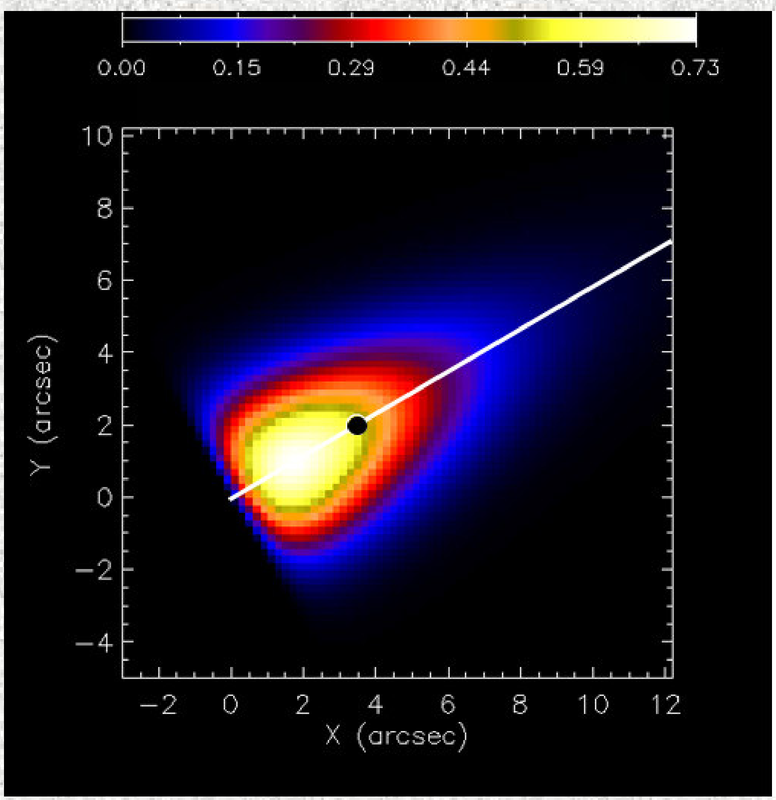
Centroid from visibilities

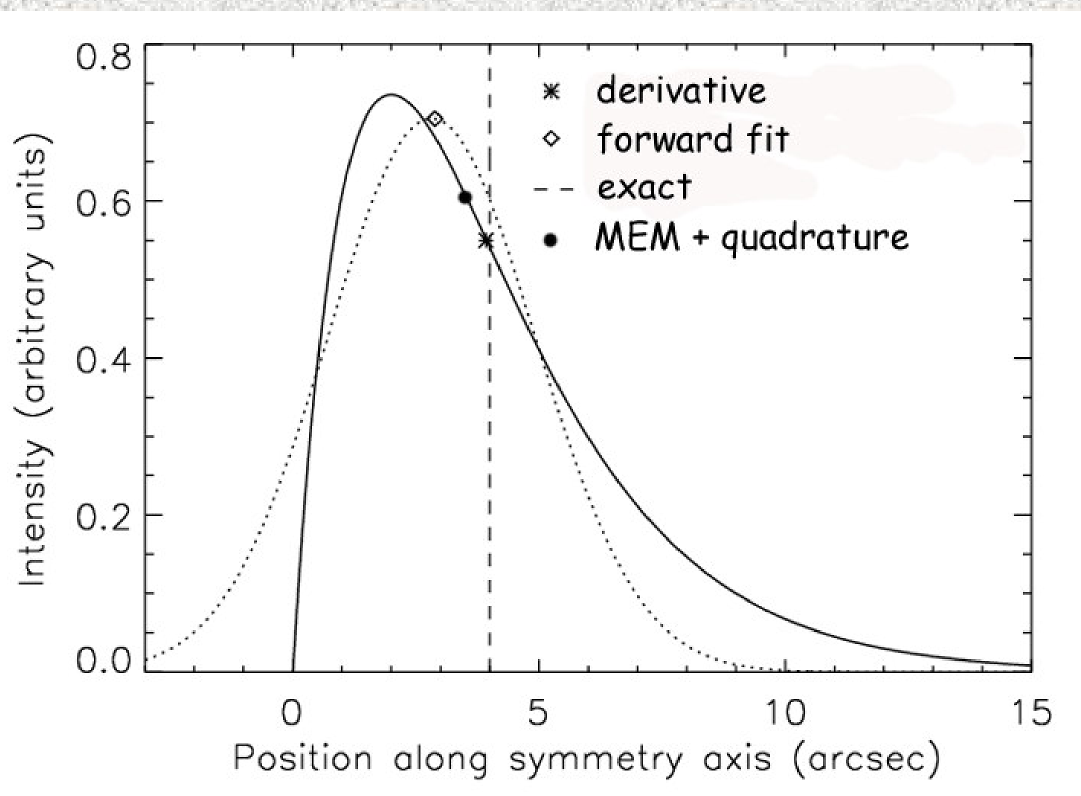


Comparison

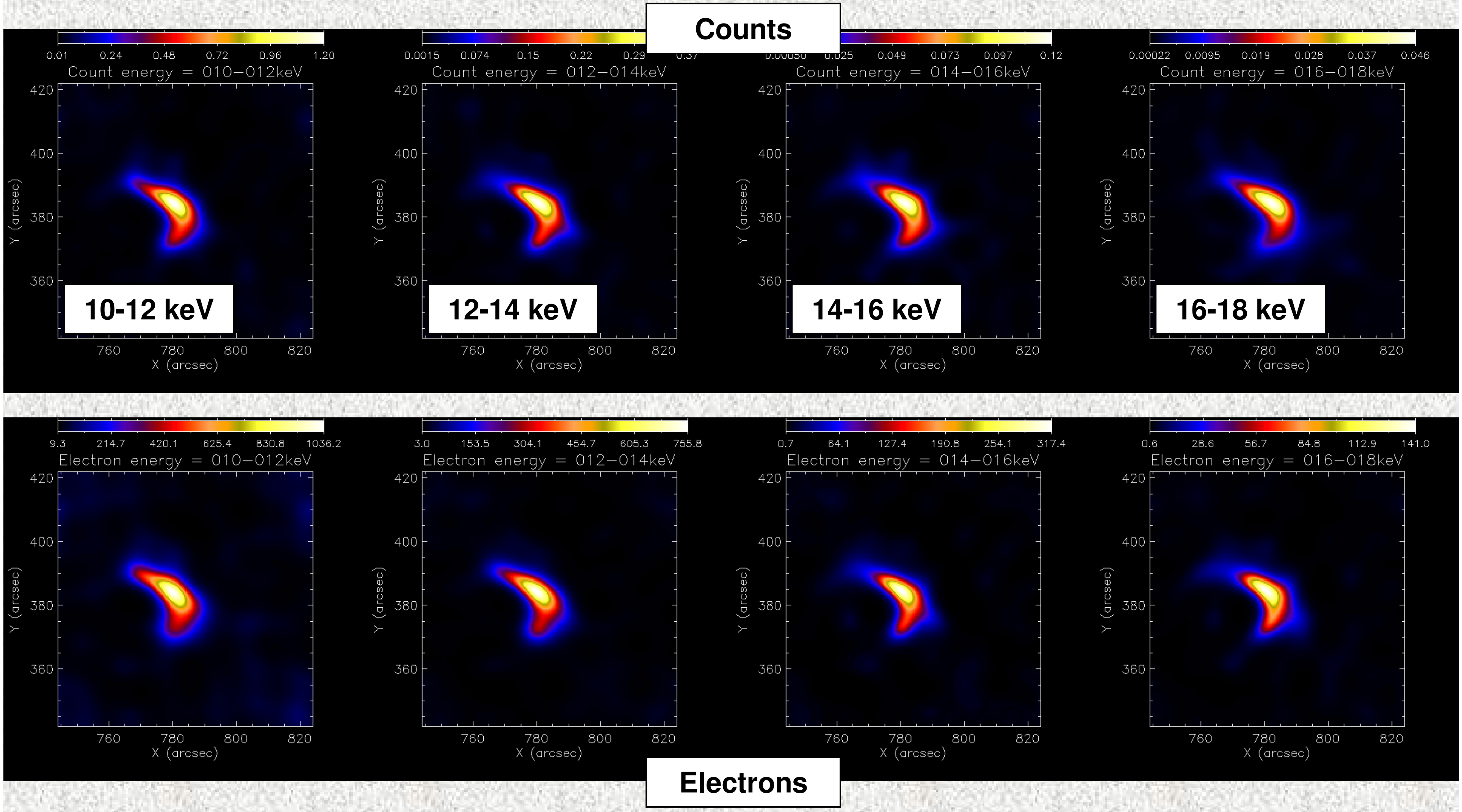
Theory: $(x_c , y_c) = (3.464 , 2.000)$ Fwdfit (ellipse): $(x_c , y_c) = (2.443 , 1.410)$

Derivative: $(x_c , y_c) = (3.364 , 1.957)$ MEM + quadr: $(x_c , y_c) = (3.036 , 1.802)$





April 15 2002, 00:05:00-00:10:00 UT



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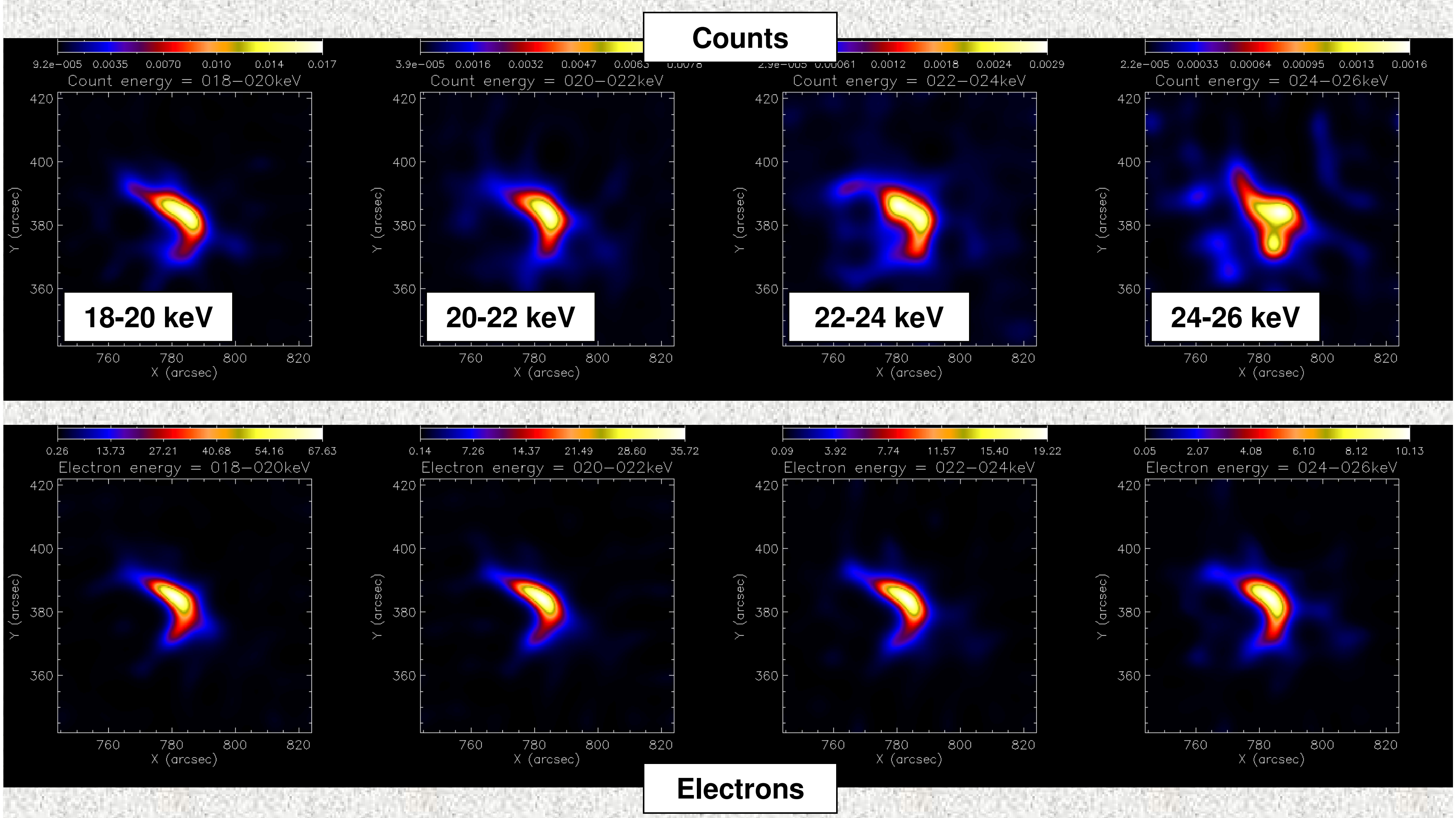

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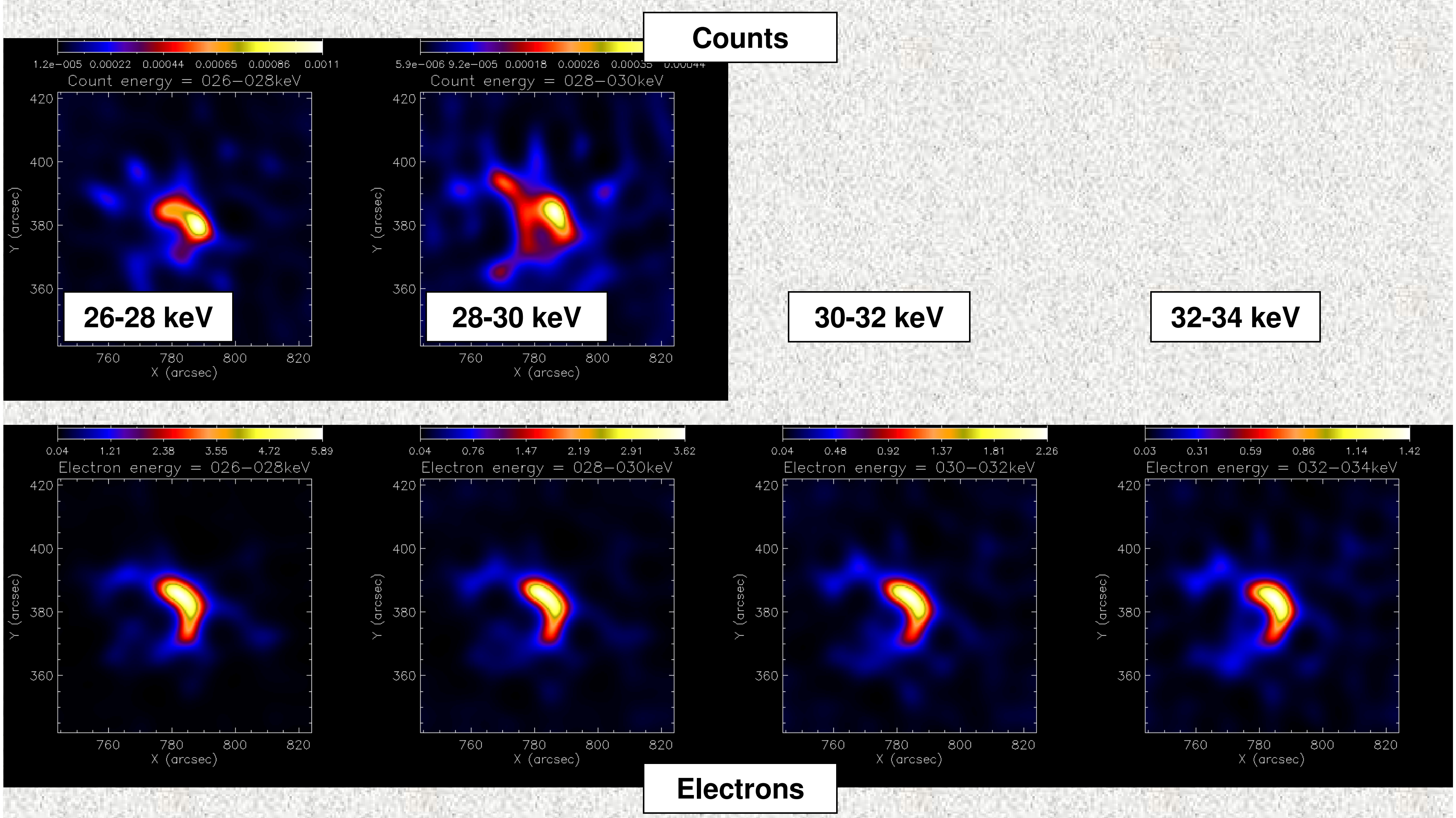


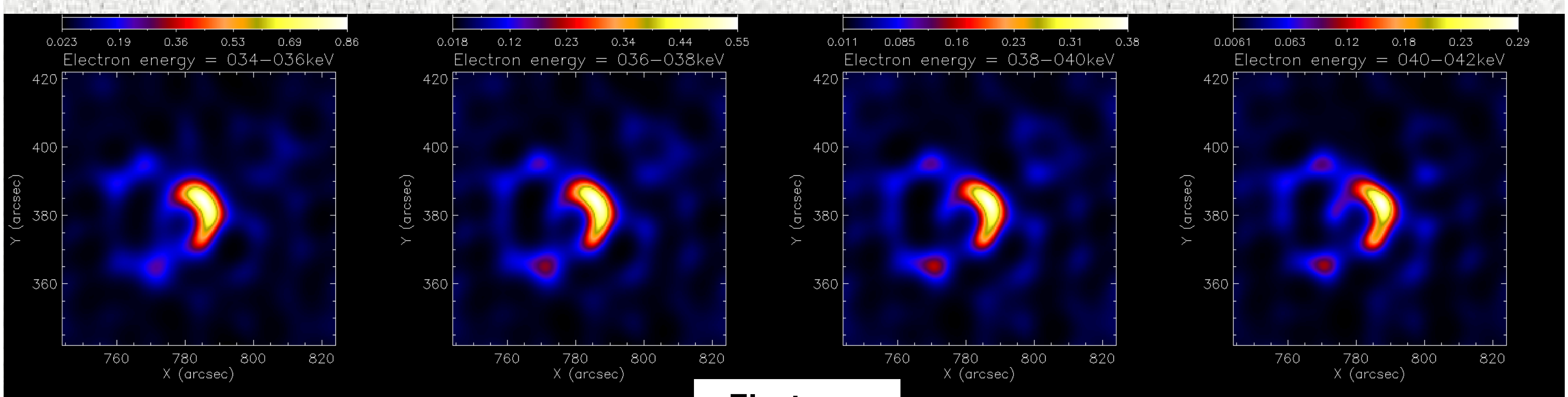
April 15 2002, 00:05:00-00:10:00 UT

34-36 keV

36-38 keV

38-40 keV

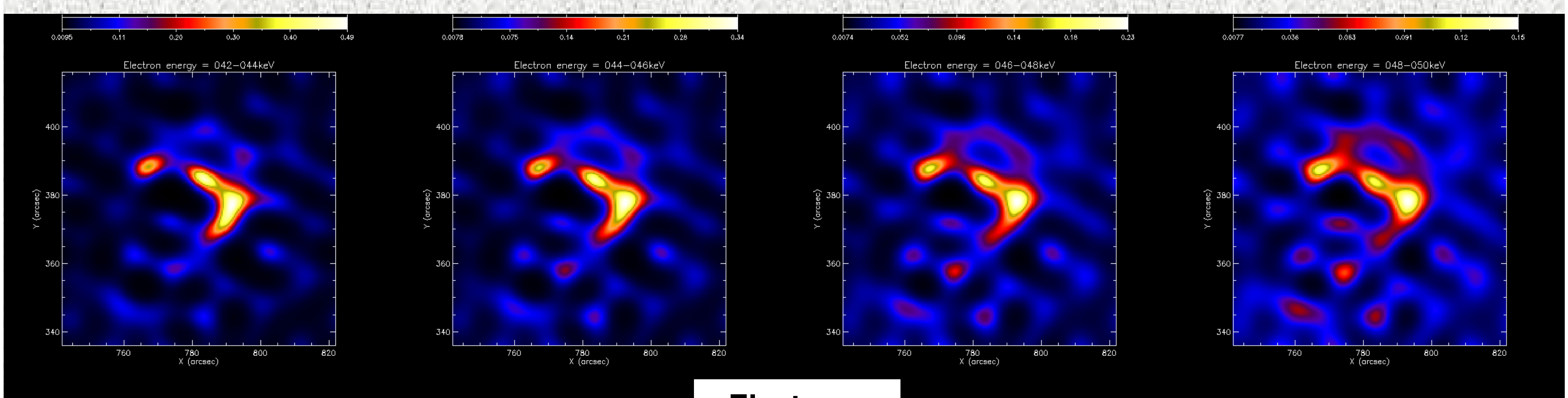
40-42 keV



Electrons

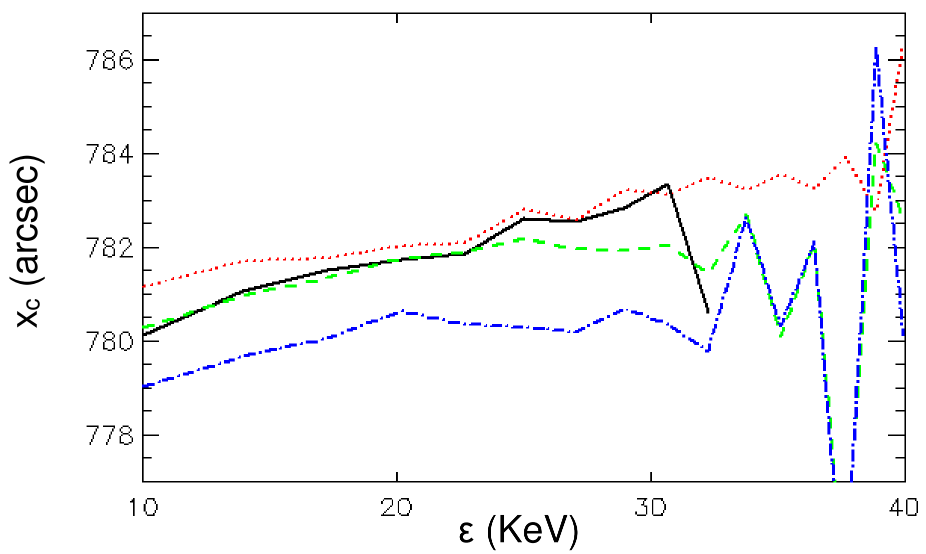
April 15 2002, 00:05:00-00:10:00 UT

42-44 keV | **44-46 keV** | **46-48 keV** | **48-50 keV**



Electrons

X-ray centroids



X-ray energies

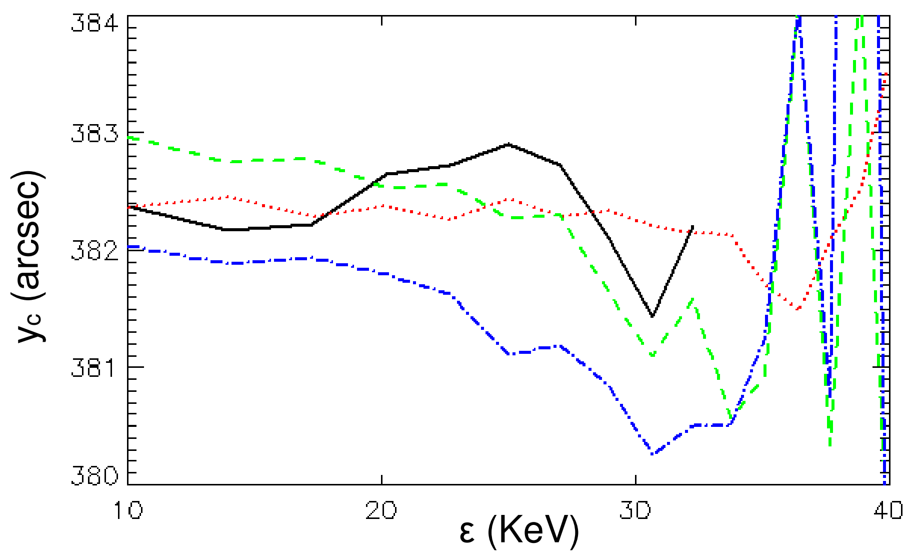
~ 10-40 keV

Solid: MEM + quadrature

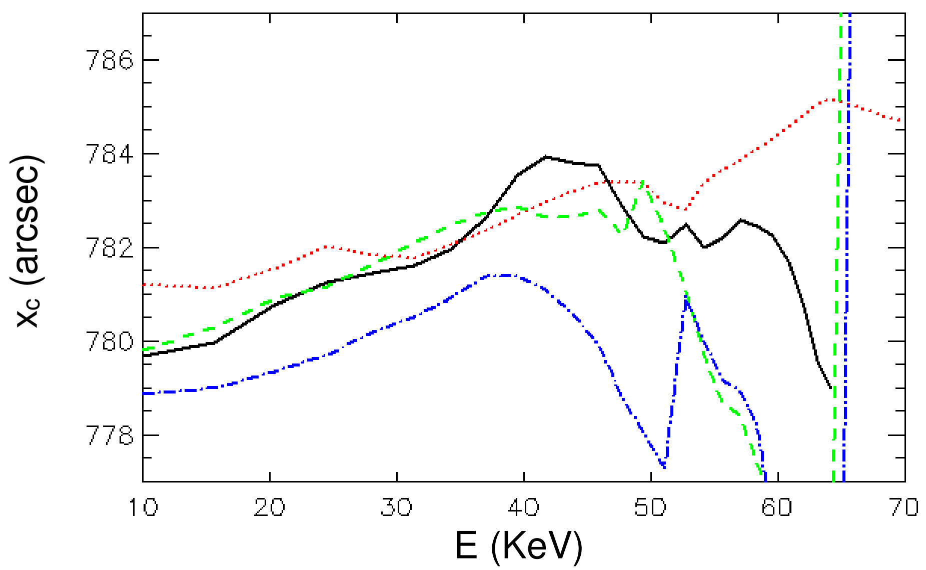
Dotted: derivative

Dashed: forward fit (ellipse)

Dot dashed: forward fit (loop)



Electron centroids



Electron energies

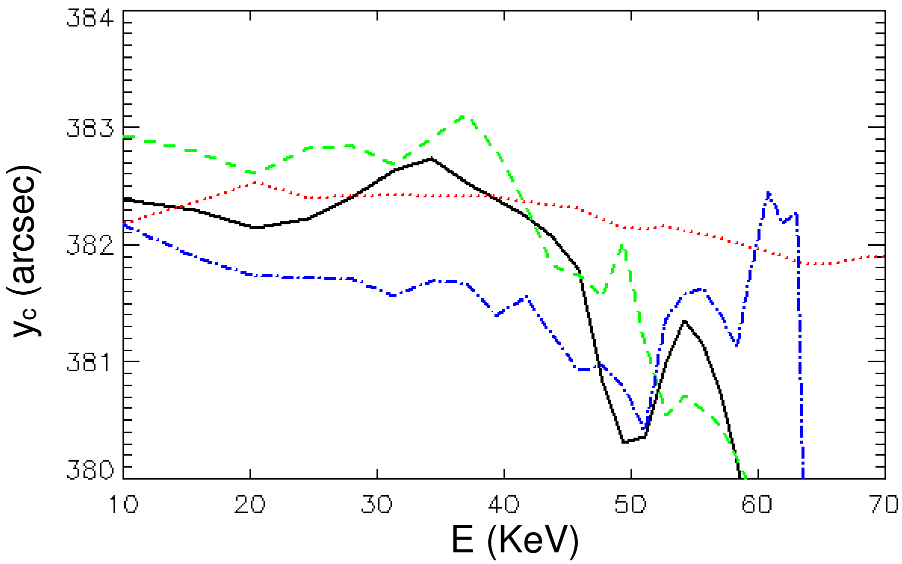
~ 10-70 keV

Solid: MEM + quadrature

Dotted: derivative

Dashed: forward fit (ellipse)

Dot dashed: forward fit (loop)



Conclusions

- New method to get the flare location starting from the visibilities

- No need to assume shapes or construct maps

- Combined with regularized inversion, can be extended to the electron space

- Further work needed to make the method robust and automatic

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