The electron-acoustic mode

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This paper examines electrostatic modes in an unmagnetized, homogeneous, Vlasov plasma with three Maxwellian components: ions, hot electrons, and cool electrons. In such a plasma, the electron-acoustic mode with frequencies between the ion and electron plasma frequencies may propagate with light damping. The conditions that allow propagation of this mode, which is distinct from the well-known ion-acoustic and Langmuir waves, are given in detail; approximate necessary conditions are $10 \lesssim T_h/T_c$ and $0 < n_c < 0.8 n_e$, where the subscripts *c*, *h*, and *e* refer to the cool and hot electron components and the total electron population, respectively.

I. INTRODUCTION

Over 20 years ago, Fried and Gould[1] obtained numerical solutions of the linear electrostatic Vlasov dispersion equation in an unmagnetized, homogeneous plasma. They showed that, in addition to the weakly damped, now well-known Langmuir and ion-acoustic waves, there are two families of heavily damped acoustic-like ($\omega_r \sim k$) solutions of the dispersion equation. (We reserve the term "modes" for weakly damped roots of the dispersion equation.) Although the electron acoustic-like solutions with phase speeds greater than the electron thermal speed have been given passing attention in the literature,[2,3] Montgomery's[4] comment that "they have not played an important role in any theories or experiments to date" still seems to be true, at least with respect to plasmas with a single electron component.

However, electron-acoustic modes can become important if the plasma electrons can be described in terms of two components, one hot (denoted by the subscript h) and the other relatively cold (subscript c). Under such conditions, Watanabe and Taniuti[5] used the linear electrostatic Vlasov dispersion equation to argue that the electron-acoustic mode is not strongly damped. Yu and Shukla[6] used fluid approximations to derive the dispersion of an electron-acoustic-like mode in a plasma with two electron components, although they imposed the difficult conditions $T_c/T_i \ll m_e/m_i$. In the space plasma context, theoretical calculations modeling electron distributions observed in the earth's bow shock[7] and the auroral magnetosphere[8,9] have demonstrated that a relative drift between two electron components with disparate temperatures can yield growing waves at intermediate frequencies of $\omega_i < \omega_r < \omega_e$, where ω_i (ω_e) is the ion (electron) plasma frequency and ω_r is the wave frequency. The authors of Refs. 7 and 9 have termed this mode the "electron-acoustic instability." However, there is as yet no published research describing the parametric dependence of either the electron-acoustic mode or its associated instability. It is the purpose of this communication to examine the electron-acoustic mode in a plasma with two Maxwellian electron components and to establish the parameter regime for which it is lightly damped. The properties of the unstable mode will be described elsewhere.

II DEFINITIONS AND NOTATION

We consider a charge-neutral, homogeneous, collisionless plasma bearing no current and with no magnetic field. The following symbols are defined for the jth species or component: the plasma frequency $\omega_j \equiv (4\pi n_j e_j^2/m_j)^{1/2}$, the thermal speed $v_j \equiv (T_j/m_j)^{1/2}$, and the Debye wavenumer $k_j \equiv (4\pi n_j e_j^2/T_j)^{1/2}$. The Boltzmann factor is understood to multiply the temperatures T_j throughout.

All fluctuating quantities are assumed to vary in time as $\exp(-i\omega t)$ where a complex frequency $\omega = \omega_r + i\gamma$ is assumed. Fourier transforming all fluctuating quantities in space and assuming electrostatic fluctuations leads to the usual linear dispersion equation,

$$1 + \sum_j K_j(\mathbf{k},\omega) = 0, \tag{1}$$

where $K_j(\mathbf{k},\omega)$ is the susceptibility of the jth species. In the specific case that the jth species zeroth-order distribution function is a Maxwellian,[1]

$$K_j(\mathbf{k},\omega) = -(k_j^2/2k^2)\, Z'(\zeta_j), \tag{2}$$

where $\zeta_j \equiv \omega/\sqrt{2} k v_j$. All results of this paper are obtained from exact computer solutions of Eq. (1) using Eq. (2). Throughout this paper we consider one ion component (subscript i) and two electron components, one hot (subscript h) and one cold (subscript c). Subscript e will denote overall electron properties, e.g., $n_e = n_i$.

III. RESULTS

Figure 1 illustrates the dependence on T_h/T_c of the dispersion curves for the Langmuir mode and the electron-acoustic mode in a plasma with $n_c = n_h$. At $T_h = T_c$, the well-known results of a weakly damped Langmuir wave and a strongly damped acoustic-like solution are shown. As the temperature of the hot component is increased, the phase speed at the Langmuir wave increases, as does its damping at fixed wavenumber. In contrast, the damping of the acoustic-like solution decreases at intermediate wavenumbers so that, at $T_h = 100 T_c$, this branch becomes a lightly damped mode ($|\gamma| \leqslant \omega_r/2\pi$). Following earlier references[5,7,9] we term this the "electron-acoustic mode."

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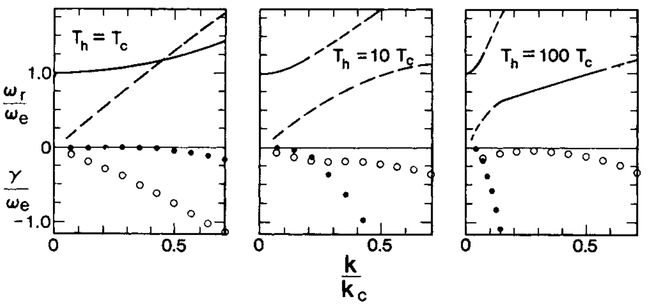

FIG. 1. The complex frequency as a function of wavenumber for the Langmuir mode and the electron-acoustic mode in a plasma with two electron components for three different values of T_h/T_c. Here, $n_h = n_c = 0.50n_e$. Here ω_r is indicated by the solid lines (at $|\gamma| \leqslant \omega_r/2\pi$) and dashed lines (corresponding to $|\gamma| > \omega_r/2\pi$), whereas γ is represented by the solid dots (corresponding to the Langmuir mode) and the open circles (the electron acoustic mode). Here and in all the figures, $m_i = 1836m_e$ and $T_c/T_i = 1.0$.

As the rightmost panel of Fig. 1 shows, there are actually three regimes for the stable electron-acoustic mode at $T_h \gg T_c$: an acoustic regime $\omega_r \sim k$ at small k which is heavily damped, a lightly damped cold plasma regime $\omega_r \sim \omega_c$ at intermediate wavenumber, and another heavily damped regime at $\omega_r > \omega_c$ and relatively short wavelengths. Although in the present model this mode is heavily damped in the acoustic regimes, the addition of a relative drift between the two electron components leads to wave growth in both the acoustic and cold plasma regimes.[9] We regard this as sufficient justification for calling this mode "electron acoustic."

Since $\omega_r \gg \omega_i$ in general, the ions are nonresonant ($|\zeta_i| \gg 1$) for all nonzero k, and the changes in dispersion properties are caused by changes in the response of the electron components. In the long-wavelength acoustic regime, $|\zeta_c| \gg 1$ but $|\zeta_h| \lesssim 1$, and there is strong Landau damping by the hot component. Here
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$$\omega_r \simeq (n_c/n_h)^{1/2} k v_h .$$

Increasing k corresponds to a decreasing phase speed and ζ_h so that Landau damping by this species decreases. As long as T_h/T_c is sufficiently large, there is a nontrivial range of wavenumbers at which $|\zeta_h| \ll 1$ and $|\zeta_c| \gg 1$ so that Landau damping is weak and the mode may propagate. In this regime,

$$\omega_r^2 \simeq \omega_c^2 \frac{1 + 3(k^2/k_c^2)}{1 + k_h^2/k^2} .$$

As k increases further, ζ_c becomes of order unity and damping on the cold electrons quenches the mode.

Figure 2 illustrates how the dispersion of the Langmuir mode and the electron-acoustic mode changes at $T_h = 100T_c$, as the relative densities of the two electron components are varied. The electron-acoustic mode as described above exists over a broad range of cold-electron densities ($0.20 \leqslant n_c/n_e < 0.80$). However, as the cold-plasma density gets larger and ω_c approaches ω_e, the lightly damped mode changes character and becomes a cold-plasma mode which makes contact with the Langmuir mode at small wavenumber.

Figure 3 illustrates curves of constant γ/ω_r in the wavenumber versus T_h/T_c and real frequency versus T_h/T_c

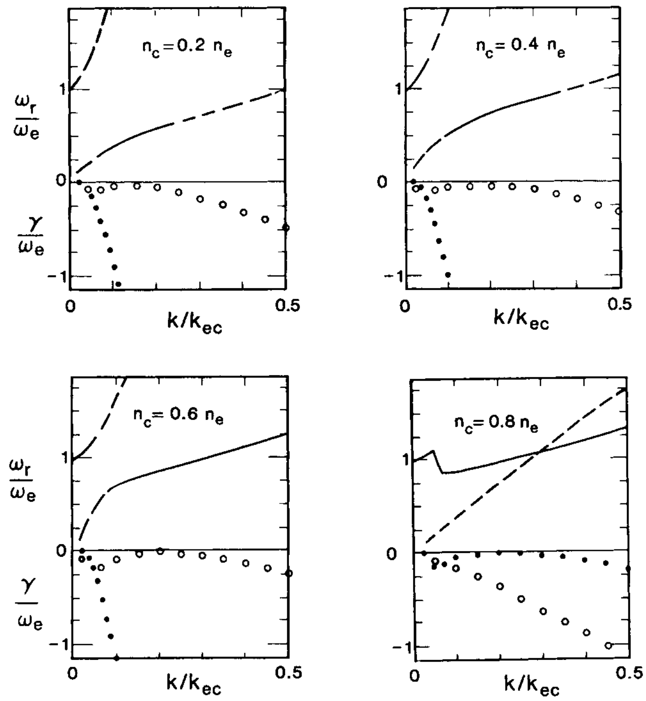

FIG. 2. The complex frequency of two modes as a function of wavenumber for four values of the cold-electron component density. The solid/dashed lines represent ω_r of the Langmuir-like mode (ω_r approaches ω_e as k becomes small) and the electron-acoustic mode (ω_r approaches zero as k becomes small); the lines of solid dots and open circles represent γ of the Langmuir-like mode and the electron acoustic mode, respectively. The solid/dashed line convention is the same as in Fig. 1. Here $T_h = 100T_c$ and $k_{ec}^2 = 4\pi n_e e^2/T_c$.

planes. This figure shows the threshold value of T_h/T_c necessary for the electron-acoustic mode to exist. In addition, Fig. 3 shows how this mode is confined to a relatively narrow range of wavenumbers ($0 < k < 0.6k_c$ for $T_c = T_i$) and frequencies ($\omega_c \lesssim \omega_r \lesssim \omega_e$) out to very large T_h/T_c values.

Figure 4 summarizes the results of this paper by illustrating the regime of the electron-acoustic mode in n_c/n_e vs T_h/T_c parameter space. Three criteria were used to define this regime: (1) the mode is lightly damped at nonzero wavenumber, $|\gamma| \leqslant \omega_r/2\pi$, (2) the root is acoustic-like at small k, and (3) the frequency lies in the intermediate range

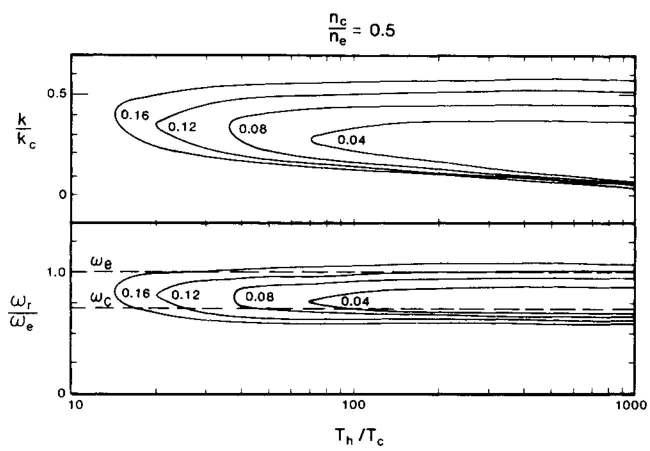

FIG. 3. Curves of constant $|\gamma|/\omega_r$ in the wavenumber versus T_h/T_c and real frequency versus T_h/T_c planes. Here $n_h = n_c = 0.50n_e$.

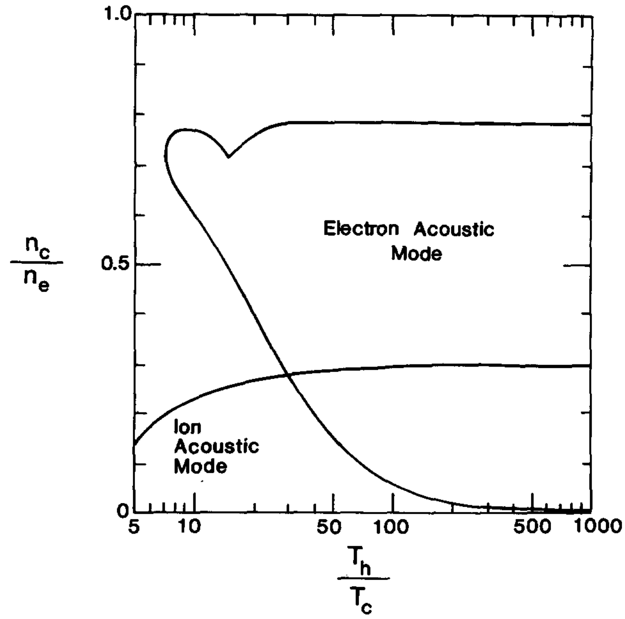

FIG. 4. The weakly damped ($|\gamma| < \omega_r/2\pi$) regimes of the electron acoustic mode (acoustic-like dispersion at small wavenumbers and $\omega_i < \omega_r < \omega_e$) and the ion-acoustic mode ($\omega_r \lesssim \omega_i$) in the parameter space of cold-plasma density versus hot-to-cold electron component temperature ratio.

$\omega_i < \omega_r < \omega_e$. The small damping criterion generally corresponds to the lower part of the curve ($n_c/n_e \lesssim 0.60$) so that the region below this line corresponds to a heavily damped electron-acoustic mode. The criterion $\omega_r < \omega_e$ is generally not satisfied in that region which lies above the curve and above about $n_c = 0.70 n_e$, so that in this regime the mode becomes Langmuir-like (e.g., the fourth panel of Fig. 2).

Also shown in Fig. 4 is the lightly damped regime of the ion-acoustic mode for $T_c = T_i$. As one might expect, this mode propagates only when the hot component is sufficiently dense ($0.30 n_e \lesssim n_h$) and sufficiently hot ($5T_c \lesssim T_h$). Of course, the Langmuir mode is lightly damped at long wavelengths for all parameters of Fig. 4. The utility of Fig. 4 is that it readily illustrates the parametric regime in which the electron-acoustic mode can sustain electrostatic fluctuations at intermediate frequencies.

ACKNOWLEDGMENTS

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