Resonance in a Plasma with Two Ion Species

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When a high-density plasma column in an axial magnetic field possesses two (or more) ion species of different charge-to-mass ratios, there exists a plasma resonance condition which involves only the ion cyclotron frequencies. At reasonance, the two ion clouds oscillate transversely to the static magnetic field and 180 deg out of phase with each other, while the electrons remain relatively motionless. The ratio of the ion oscillatory energy to that of the electrons is of the order of the ratio of the ion-to-electron masses. Collisions between the two ion clouds randomize the large ordered velocities of the ions with great efficiency. Thus, by exciting this resonance, considerable ion heating may be realized. The effect of varying the relative concentration of the two ions is discussed.

IT is the purpose of this paper to point out that when a high-density plasma column coaxial with a static magnetic field, possesses two (or more) ion species with different charge-to-mass ratios, there exists a plasma resonance condition which involves only the ion cyclotron frequencies. As will be shown later, at resonance, the two ion clouds oscillate with large amplitudes 180 deg out of phase with each other while the electrons remain relatively motionless. By exciting this resonance by an external circuit, it may be possible to heat the ions in the plasma preferentially over the electrons.

The axially symmetric, transverse oscillations of a long plasma column in an axial magnetic field are accompanied by time varying electric and magnetic fields whose geometrical configuration is similar to that of the fields of the extraordinary plane wave[1] in a uniform plasma.[2,3]

Several writers[3-6] have shown that the propagation constant k_x of such a wave in a cold collisionless plasma, possessing only a single ion species, is given by

$$k_x^2 = k_0^2\left[1 + \langle\omega_p^2\rangle \cdot \frac{\langle\omega_p^2\rangle - (\omega^2 - \omega_b\omega_{b+})}{(\omega^2 - \omega_b^2)(\omega^2 - \omega_{b+}^2) - \langle\omega_p^2\rangle(\omega^2 - \omega_b\omega_{b+})}\right], \quad (1)$$

where $k_0 = \omega/c$ is the propagation constant in free space, $\langle\omega_p^2\rangle = (M + m)\omega_p^2/M$ is the total plasma frequency squared, ($\omega_p^2 = ne^2/m\epsilon_0$ is the electron plasma frequency squared), $\omega_b = eB_0/m$ is the electron cyclotron frequency, and $\omega_{b+} = eB_0/M$ is the ion cyclotron frequency. The propagation constant k_x exhibits a resonance (defined by $k_x = \infty$) at a frequency ω which is a root of

$$D(\omega) = (\omega^2 - \omega_b^2)(\omega^2 - \omega_{b+}^2) - \langle\omega_p^2\rangle(\omega^2 - \omega_b\omega_{b+}) = 0. \quad (2)$$

Equation (2) has two positive roots which to a good approximation are given by

$$\omega_1^2 = \omega_b^2 + \omega_p^2 \quad (3a)$$

and

$$\omega_2^2 = \omega_b\omega_{b+}\left(\frac{\omega_b\omega_{b+} + \omega_p^2}{\omega_b^2 + \omega_p^2}\right). \quad (3b)$$

These are plotted in solid lines in Fig. 1 as a function of the normalized electron density ω_p^2/ω_b^2. We note that the upper branch of the resonant frequency (given by ω_1) is sensibly independent of the ion cyclotron frequency. The reason for this lies, of course, in the smallness of the electron mass. Consequently, at the high frequencies given by Eq. (3a), the electrons, because of their large mobilities, carry almost all of the current. The lower branch (given by ω_2) gives a resonance at the ion cyclotron frequency but only for plasma densities which are sufficiently low that $\omega_p^2 \ll \omega_b\omega_{b+}$. At high plasma densities ($\omega_p^2 > \omega_b^2$) sufficient charge separation occurs that the component of the electric field which rotates with the ions (and which is responsible for the absorption at the ion cyclotron frequency at low densities) is shielded out.[7,8] At the

[1] The electric field vector $\mathbf{E}$ and the propagation constant $\mathbf{k}_x$ of the extraordinary wave are both at right angles to the static magnetic field $\mathbf{B}_0$, but $\mathbf{E}$ and $\mathbf{k}_x$ are not necessarily at right angles to each other.
[2] K. Körper, Z. Naturforsch. **a12**, 815 (1957).
[3] P. L. Auer, H. Hurwitz, Jr., and R. D. Miller, Phys. Fluids 1, 501 (1958).
[4] E. Astrom, Arkiv Fysik. **2**, 443 (1950).
[5] L. Mower, Tech. Rept. No. MPL-1 Sylvania Electric Products, Inc. (1956).
[6] W. P. Allis and S. J. Buchsbaum, Notes on Plasma Dynamics, Summer Session, MIT, 1959, Sec. B (unpublished).
[7] An ingenious way out of this difficulty was found by Stix [T. H. Stix and R. W. Palladino, Phys. Fluids 1, 446 (1958)] who operates alternate sections of an induction coil 180° out of phase with each other, thus allowing the radial space charge to neutralize by electron motion along the lines of the magnetic field. This is tantamount to exciting plasma oscillation with a finite wavelength in the axial direction.
[8] T. H. Stix, Phys. Rev. **106**, 1146 (1957).

high plasma densities there is a resonance at $\omega_2 = (\omega_b\omega_{b+})^{\frac{1}{2}}$ sometimes called the "hybrid" resonance,[3] which is associated with the large charge separation that causes the E field to be nearly longitudinal. An examination of the equations of motion for the electrons and ions leads to the following results for electron and ion velocities at the $\omega_2^2 = \omega_b\omega_{b+}$ limit:

$$(M - m)\mathbf{V}_+ = -j(Mm)^{\frac{1}{2}}\mathbf{E}/B_0 + m\mathbf{B}_0 \times \mathbf{E}/B_0^2, \quad (4a)$$

$$(M - m)\mathbf{v}_- = -j(Mm)^{\frac{1}{2}}\mathbf{E}/B_0 - M\mathbf{B}_0 \times \mathbf{E}/B_0^2, \quad (4b)$$

where $\mathbf{V}_+$ and $\mathbf{v}_-$ are the ordered ion and electron velocities, and $\mathbf{E}$ is the electric field component of the electromagnetic field. It is seen that electrons and ions move together along $\mathbf{E}$ but their motions are in opposite phase at right angles to $\mathbf{E}$ ($\mathbf{E}$ being along the wave normal) the amplitude of the electron velocity being M/m times that of the ion velocity. Even though the kinetic energies of the ordered motion of the electrons and ions are of the same order of magnitude, electron-ion collisions in a fully ionized plasma or electron-atom and ion-atom collisions in partially ionized plasma will randomize the electron velocity at a larger rate than the ion velocity. It is for this reason that the hybrid resonance is not useful either as a diagnostic tool for the study of ion processes in a plasma or as a method for heating the ions.

However, when the plasma consists of two ion species (assumed positive), there will exist a third resonance branch (dotted line in Fig. 1) which depends predominantly on the motion of the ions and not of the electrons. Let M_1 and M_2 be the respective masses of the two ion species, x_1 and x_2 their relative concentrations ($x_1 + x_2 = 1$), and ω_{b1}

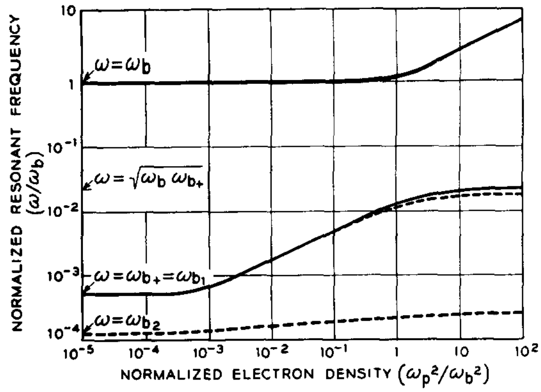

FIG. 1. Resonant frequency of the extraordinaty wave as a function of the normalized electron density. The solid line represents the resonance in a plasma with a single ion species with an ion-to-electron mass ratio of 2000:1. The dashed line represents the resonance in a plasma with two singly ionized ion species with electron-to-ion mass ratios of 1:2000:8000. The plot is for a relative ion concentration of 50%.

and ω_{b2} their cyclotron frequencies ($\omega_{bi} = e_iB/M_i$). The propagation constant k_x is best obtained from the formula[5]

$$\tfrac{1}{2}k_x^2 = \frac{k_r^2k_l^2}{k_r^2 + k_l^2}, \quad (5)$$

where k_r and k_l are the propagation constants of right- and left-handed circularly polarized waves propagating *along* the static magnetic field. In the absence of collisions, the constants k_r and k_l are given by[4]

$$\frac{k_r^2}{k_0^2} = 1 - \frac{\omega_p^2}{\omega}\left[\frac{1}{\omega - \omega_b} + \frac{x_1f_1}{\omega + \omega_{b1}} + \frac{x_2f_2}{\omega + \omega_{b2}}\right], \quad (6)$$

$$\frac{k_l^2}{k_0^2} = 1 - \frac{\omega_p^2}{\omega}\left[\frac{1}{\omega + \omega_b} + \frac{x_1f_1}{\omega - \omega_{b1}} + \frac{x_2f_2}{\omega - \omega_{b2}}\right],$$

where $f_i = m/M_i$.

From Eqs. (5) and (6) we find

$$\frac{k_x^2}{k_0^2} = 1 + \omega_p^2\,\frac{\omega_p^2[\omega^2 - (x_2\omega_{b1} + x_1\omega_{b2})^2] - [\omega^2 - \omega_b(x_1\omega_{b1} + x_2\omega_{b2})]\left[\omega^2 - \dfrac{x_2f_1 + x_1f_2}{x_1f_1 + x_2f_2}\,\omega_{b1}\omega_{b2}\right]}{(\omega^2 - \omega_b^2)(\omega^2 - \omega_{b1}^2)(\omega^2 - \omega_{b2}^2) - \omega_p^2[\omega^2 - \omega_b(x_1\omega_{b1} + x_2\omega_{b2})]\left[\omega^2 - \dfrac{x_2f_1 + x_1f_2}{x_1f_1 + x_2f_2}\,\omega_{b1}\omega_{b2}\right]}, \quad (7)$$

where, in order to simplify the expression, we neglected f_i compared with unity. Equation (7) has a form similar to Eq. (1) [it reduces properly to Eq. (1) if either x_1 or x_2 are zero]. At low plasma densities it exhibits a resonance at each of the three cyclotron frequencies eB_0/m, eB_0/M_1, and eB_0/M_2. At large plasma densities, of the three resonances two involve both electrons and ions, $\omega \simeq \omega_p$ and $\omega^2 \simeq \omega_b(x_1\omega_{b1} + x_2\omega_{b2})$, and one resonance at

$$\omega^2 = \frac{x_2f_1 + x_1f_2}{x_1f_1 + x_2f_2}\,\omega_{b1}\omega_{b2} \quad (8)$$

involves only the ions. The resonance given by Eq. (8) is also of the "plasma resonance" type in that it is accompanied by a large amplitude of the ac space charge. Here, however, an examination of the equations of motion reveals that when Eq. (8) is satisfied, the electron and ion velocities are given by

$$\mathbf{v}_- = -j\,\frac{m}{M_{\mathrm{eff}}}\frac{\mathbf{E}}{B_0} - \frac{\mathbf{B}_0 \times \mathbf{E}}{B_0^2},$$

$$\mathbf{V}_1 = \frac{M_1M_{\mathrm{eff}}}{M_{\mathrm{eff}}^2 - M_1^2}\left(j\,\frac{\mathbf{E}}{B_0} + \frac{M_{\mathrm{eff}}}{M_1}\frac{\mathbf{B}_0 \times \mathbf{E}}{B_0^2}\right),$$

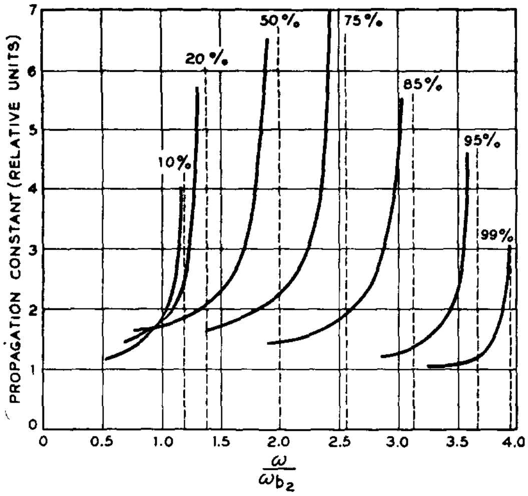

FIG. 2. Propagation constant of the extraordinary wave as a function of frequency near the ion cyclotron frequencies of the two ions for various relative concentrations of the heavier ion. Ion mass ratio = 4. Only one-half of the dispersion curve is plotted.

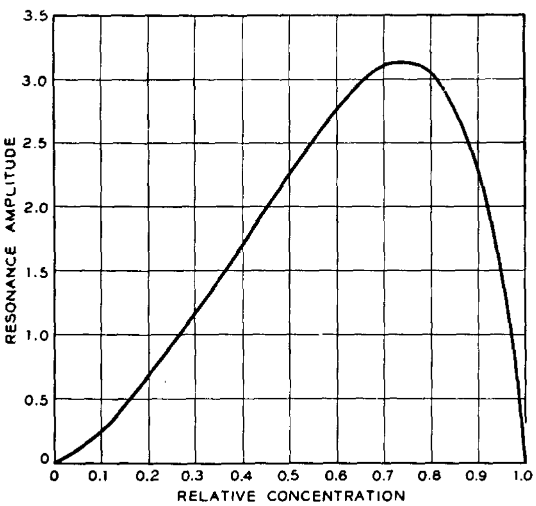

FIG. 3. Magnitude of the resonance as a function of the relative concentration of the heavier ion. Ion mass ratio = 4.

$$\mathbf{V}_2 = \frac{M_2 M_{\text{eff}}}{M_{\text{eff}}^2 - M_2^2}\left(j\frac{\mathbf{E}}{B_0} + \frac{M_{\text{eff}}}{M_2}\frac{\mathbf{B}_0 \times \mathbf{E}}{B_0^2}\right),$$

where M_{eff} is an effective mass defined by

$$M_{\text{eff}}^2 = M_1 M_2 \frac{x_1 f_1 + x_2 f_2}{x_2 f_1 + x_1 f_2}.$$

We note that the ion velocities along $\mathbf{E}$ are large compared to electron velocities and are out of phase with each other (since $M_1 \leq M_{\text{eff}} \leq M_2$ if we take

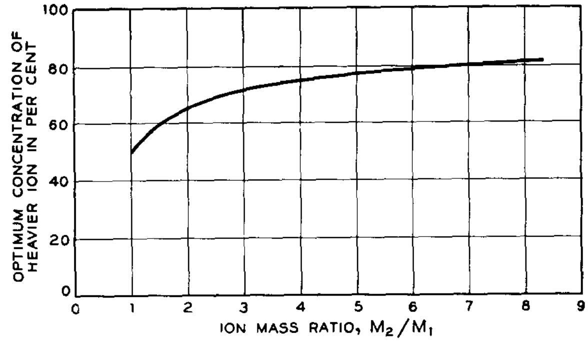

FIG. 4. Optimum concentration of the heavier ion as a function of the ion mass ratio.

M_2 to be the mass of the heavier ion). Since at resonance only a small amount of collisional damping leads to large absorption of the wave, considerable ion heating may be expected.

Both the magnitude and position of the resonance are a function of the ratio of the ion masses (the ions are assumed singly charged) and of their relative concentration. The relation between frequency and concentration is given by Eq. (8). To obtain an idea of the shape and magnitude of the resonance, Eq. (7) is plotted in Fig. 2 for an assumed mass ratio of 4 and for various ion concentrations. The plot and numerical calculations indicate that for this mass ratio the magnitude of the resonance (defined as the difference between the frequencies at which k_x^2/k_0^2 is infinite and is unity) is largest at a relative concentration of the heavier ion of 75%. The magnitude of the resonance as a function of the concentration is shown in Fig. 3, while the plot of optimum concentration of the heavy ion as a function of the ion mass ratio is shown in Fig. 4.

For an optimum concentration of a D-T mixture, the energies of the ordered motions of the various components divide as follows: Let U_-, U_D, U_T be the energies of electrons, deuterium, and tritium ions and m, M_D, and M_T their masses, then

$$U_D/U_- = 14.5(M_T/m),$$
$$U_D/U_T = 2.22 \simeq (M_T/M_D)^2.$$

At optimum concentration most of the energy (69%) goes into the lighter ion. However, by varying the concentration away from the optimum the relative energies can be varied at will (at the expense of decreasing the total power input into the plasma).